Thermal Science & Engineering

Law of Thermodynamics

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Laws of Thermodynamics

THERMODYNAMIC SYSTEM & SURROUNDINGS

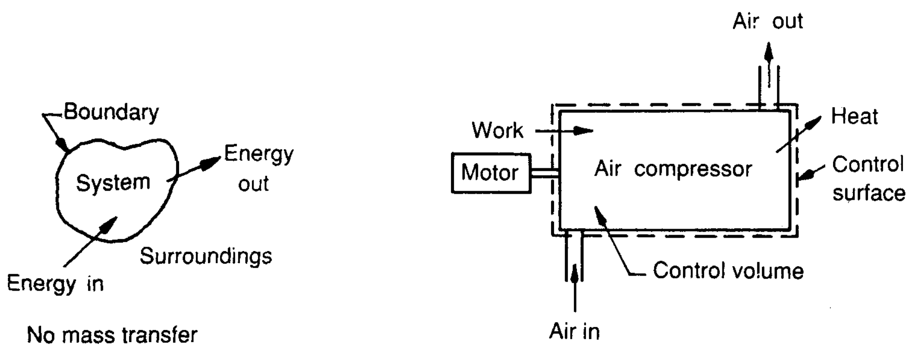
A thermodynamic system may be considered as a quantity of working substance with which heat and work interactions are studied. The envelope enclosing the system, which may be real or hypothetical is known as boundary of the system. The region outside the system is known as surroundings. The transfer of mass and energy take place between the system and surroundings.

Thermodynamic system are classified as

(a) closed (b) open (c) isolated

Closed System (Non Flow System)

A good example of closed system is a piston and cylinder as shown in figure given below:

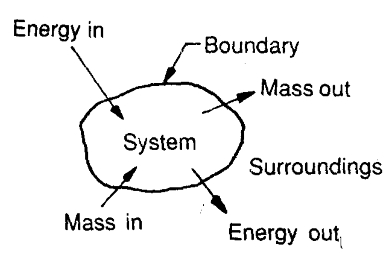


Closed systems

No mass can flow in or out, only heat or work or both may flow into and out of the closed system. If heat is supplied to the cylinder from external source, the volume of the gas increases and the piston moves up. Work transfer occurs due to the movement of the boundary of the system.

Open System (Flow System)

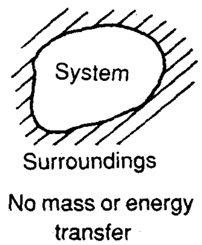
In open system, the working substances used crosses the boundary of the system. Heat and work may also cross the boundary. (see fig.)



The system turbine, gas turbine and rotary compressor are good examples of open system.

Isolated System

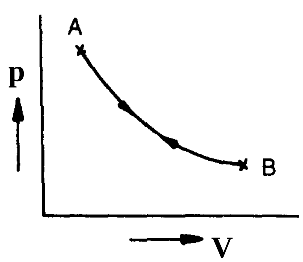
An isolated system has no mass or energy interactions with the surroundings. Though such a system has no practical interest, it is a useful concept in the study and analysis of thermodynamics principles.



An isolated system

REVERSIBLE OR IRREVERSIBLE PROCESSES

A process is said to be *reversible* if the reversal of the process does not leave any trace on the system or the surroundings. For example, if during a process from state A to B as shown in figure given below, the work and heat transfers are W and Q and if by supplying back W and Q to the system, the state of the system can be brought back from B to A, the process is said to be reversible.



If there is any change in the requirement of work and heat to bring back the system from state B to A, the process becomes *irreversible*. *The processes used in practice are mostly irreversible* due to friction, heat transfer and mixing, but in many cases idealization is used for analysis.

PRESSURE

The standard atmospheric pressure is defined as the pressure produced by a column of mercury 769 mm high. The standard atmospheric pressure is 1.0332 kgf/cm^2 and is denoted by atm. In S.I. units, it is expressed in N/m^2 or Pascal abbreviated as Pa. Various pressure units are

1 Pa = 1 N/m^2

1 bar = 10^5 Pa = 100 kPa = 0.1 MPa

1 atm = 101.325 kPa = 1.01325 bar

1 metre mercury head = 1.3366 bar

The pressure relative to a perfect vacuum is called *absolute pressure*.

Abs. pressure = gauge pressure + atmospheric pressure

When the pressure in a system is less than atmospheric pressure, the gauge pressure becomes negative, but is frequently designated by a positive number and called *vacuum*. For example, 16 mm vacuum will be

$$\frac{76-16}{76} \text{x} 1.013 = 0.80 \,\text{bar}$$

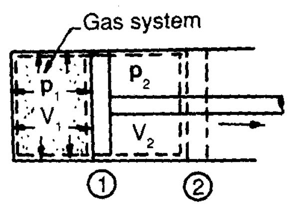
WORK, POWER & ENERGY

Work is said to be done when a force acts upon a body causing that body to move along the direction of the force. The work done given by the product of force and distance travelled by the body along the direction of the force.

W = Force (F) x Distance (S)

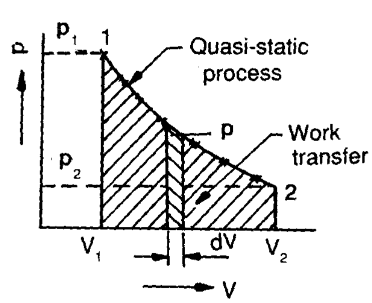
If a force of 1 N acts on a body causing 1 m displacement , the work done on the body is given by

W = 1 N x 1 m = 1 N-m



Consider a closed system of a piston and cylinder as shown in Fig. given above. The net pressure of the gas in the system causes the piston to move in the forward direction. The force acting on the piston is p.A, where A is the area of the piston, if the piston moves through a small distance dx, the work done by the gas on the piston = p.A dx = p.dv as A.dx is the change in the volume of the gas. If the gas expands from state 1 to 2 as shown in the figure, the work done is given by,

$$W = \int_{v_1}^{v_2} p.dv$$



Hence we see that work done is equal to $\int pdv$ under following conditions:

- System is closed and process takes place in non flow system
- Process is quasi static (reversible)
- Boundary of the system should move in order that work may be transferred

Power is the rate at which work is done. The common unit of power is watt (W), kilowatt (kW) or megawatt (MW).

1 W = 1 J/s = 1 Nm/s

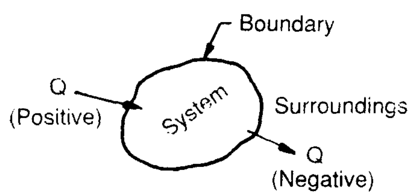
1 kW = 1000 W

Energy may exist in the form of potential energy, mechanical work or heat. The unit of energy is N-m or J in S.I. units.

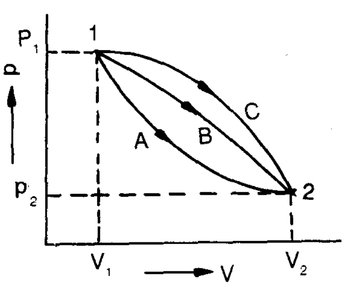
HEAT

Heat is defined as the energy transferred without transfer of mass across the boundary or a system due to a temperature difference between the system and surroundings.

The direction of heat transfer is taken from the high temperature system to the low temperature system. *Heat flow into a system is taken to be positive, and heat flow out of a system is taken as negative.*



The amount of heat transferred during a process or work done by the system during the same process is dependent on the path followed during the process. Though the end conditions are same. as shown in figure below.



The heat exchange and work don during the processes will be different for the two different paths. Thus heat and work are said to be *path functions* and *not* point functions (or properties).

The unit of heat in MKS system is *kcal* which is defined as the quantity of heat required to raise the temperature of 1 kg water through 1^{o}C. The unit of heat is S.I. system is *Joule* which is equivalent to 1 N-m.

Heat energy is generally transferred in three ways:

- **Conduction** (propagation of heat by molecular activity without actual transfer of molecules).
- **Convection** (heat transfer in fluids only by molecular collisions between hotter and cooler molecules, which lead to development of convection currents).
- **Radiation** (no medium is required for heat transfer, e.g. heating of earth surface by solar radiation).

INTERNAL ENERGY

Matter is composed of molecules which move continuously and randomly. In gases, movement of the molecules is more pronounced than in solids and liquids. Matter possesses internal kinetic energy due to the motion of its molecules. In addition to this internal K.E., matter has internal potential energy due to relative position of their molecules. The sum of these two energies is known as *specific internal energy (or simply internal energy)* and it is denoted by *u*, the unit commonly used being *J/kg* or *kJ/kg*. If the temperature of a gas is increases by adding heat, the molecular activity increases. Therefore the internal energy of a gas is a function of its temperature and its value can be increased or decreased by adding or removing heat from the gases which are commonly used as working fluids.

Total internal energy is denoted by U, its unit is J or kJ.

ENTHALPY

Enthalpy (H) of a substance at any point is quantification of energy content in it, which could be given by summation of internal energy and flow energy. Enthalpy is very useful thermodynamic property for the analysis of engineering systems.

Mathematically, it is given as,

$$H = U + PV$$

On unit mass basis, the specific enthalpy could be given as,

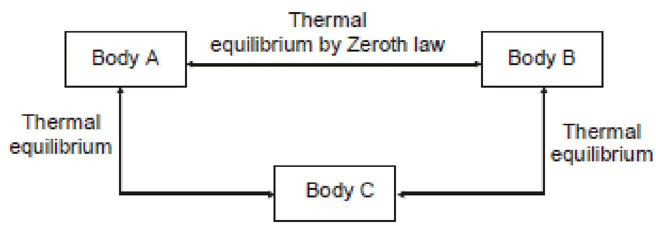
$$h = u + pv$$

Example (AMIE S08, 11, 15, 17, W16, 4 marks)

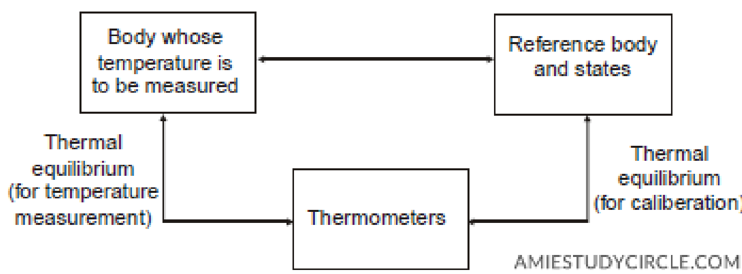
State the zeroth law of thermodynamics. Prove that this law is the basis for all temperature measurements. How the mercury in the thermometer able to find the temperature of a body using zeroth law of thermodynamics. How does the Seebeck effect make use of temperature measurement by a thermocouple?

Solution

Zeroth law of thermodynamics states that if the bodies A and B are in thermal equilibrium with a third body C separately then the two bodies A and B shall also be in thermal equilibrium with each other. This is the principle of temperature measurement. Block diagram shown in Fig. a and b show the zeroth law of thermodynamics and its application for temperature measurement respectively.



(a)



(b)

Mercury thermometer. The major significance or use of zeroth law of thermodynamics is in mercury thermometer.

When a thermometer is inserted in to the fluid, the fluid (say A) and the glass wall (say B) will attain thermal equilibrium (they are at same temperature). The glass wall (B) and mercury (say C) are at thermal equilibrium.

So A and B ; B and C are in thermal equilibrium.

This states that body A and C are in thermal equilibrium. Thus the temperature read by mercury fluid in the bulb is the temperature of the measuring fluid.

Seebeck effect. Thermo electric thermometer works on the principle of Seebeck effect. Seebeck effect says that a current flows or e.m.f. is produced in a circuit of two dissimilar metals having one junction as hot while other as cold junction. Current produced in this way is called thermo electric current while the e.m.f. produced is called thermo e.m.f. Measurement of temperature is being done by knowing the e.m.f. produced which is the thermometric property.

FIRST LAW OF THERMODYNAMICS

The law of conservation of energy states that the energy can neither be created nor destroyed. However, energy can be converted from one form to another form. The first law of thermodynamics is a particular and a more rigorous statement of this general principle with reference to energy in the form of heat and mechanical work.

The first law of thermodynamics states that heat and work are mutually convertible. It does not say anything about the possibility and method of conversion of heat into work or work into heat. It simply states that "Q" J of heat is equivalent to "W" N-m of work. This establishes a relation between work energy and heat energy. This relation was first established by Joule and the conversion factor is denoted by J and its value is 1 Newton/Joule. This simply indicates that 1 N-m mechanical work is equivalent to 1 joule of heat.

The first law of thermodynamics can be stated:

When a system undergoes a thermodynamics cycle, the net heat supplied to the system from its surroundings is equal to the net work done by the system on its surroundings.

Symbolically, the above statement is represented as dQ = dW

Energy Equation for Non-Flow System (Closed System)

Now, consider a process, during which, the internal energy of the fluid in a closed system is increased, the conservation of energy equation can be written as

Net heat supplied = gain in internal energy + Net work output

This can also be stated as

$$Q = \Delta E + W = m(u_2 - u_1) + W$$

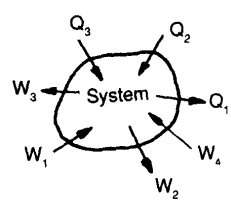
where J = 1 N-m/Joule and u_1, u_2 = internal energies in J/kg

This is applicable for non-flow reversible or irreversible process

$$Q = m(u_2 - u_1) + \int_1^2 p.dv$$

This is applicable only for non-flow reversible processes.

If there are more energy transfer quantities involved in the process, as shown in figure, the first law gives



$$Q_2 + Q_3 - Q_1 = \Delta E + (W_2 + W_3 - W_1 - W_4)$$

First Law Of Thermodynamics Applied To Open Systems

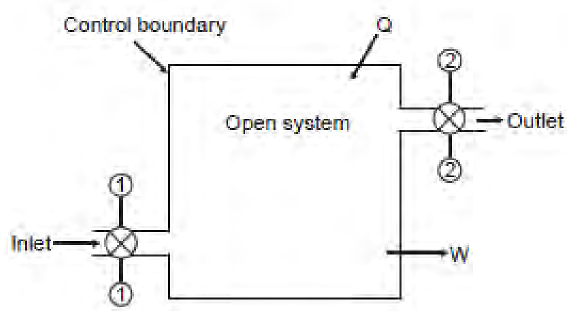
Let us consider an open system as shown in figure having inlet at section 1–1 and outlet at section 2–2.

The cross-section area, pressure, specific volume, mass flow rate, energy at section 1–1 and 2–2 are

Section 1–1 = A_1, p_1, v_1, m_1, e_1

Section 2–2 = A_2, p_2, v_2, m_2, e_2

Open system is also having heat and work interactions Q, W as shown in figure.



Applying the energy balance at the two sections, it can be given as,

Energy added to the system + Stored energy of the fluid at inlet

= Stored energy of the fluid at outlet

Quantifying the various energies;

Energy of fluid at inlet shall comprise of stored energy and flow energy as given here.

$$= m_1(e_1 + p_1v_1)$$

Similarly, energy of fluid at outlet shall comprise of stored energy and flow energy,

= Stored energy + Flow energy

$$= m_2 (e_2 + p_2v_2)$$

The energy added to the system shall be the net energy interaction due to heat and work interactions.

$$= Q - W$$

Writing energy balance, mathematically;

$$Q - W + m_1 (e_1 + p_1v_1) = m_2 (e_2 + p_2v_2)$$

or $$Q + m_1(e_1 + p_1v_1) = W + m_2(e_2 + p_2v_2)$$

If the mass flow rates at inlet and exit are same, then

$$Q + m(e_1 + p_1v_1) = W + m(e_2 + p_2v_2)$$

On unit mass basis

$$q + e_1 + p_1 v_1 = w + e_2 + p_2 v_2$$

Thus,

$$\text{Heat} + (\text{Stored energy} + \text{Flow energy})_1 = \text{Work} + (\text{Stored energy} + \text{Flow energy})_2$$

Stored energy at inlet and outlet can be mathematically given as,

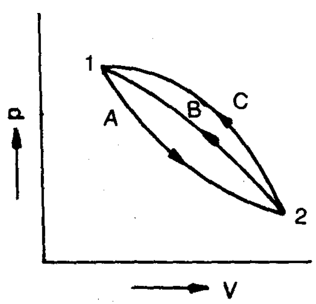
$$e_1 = u_1 + \frac{C_1^2}{2} + gz_1$$

and $$e_2 = u_2 + \frac{C_2^2}{2} + gz_2$$

where C_1 and C_2 are velocities at inlet and exit, u_1 and u_2 are internal energy at inlet and outlet, z_1 and z_2 are elevations of inlet and exit.

Energy - A property of the system

Consider following diagram



For path A $$Q_A = \Delta E_A + W_A$$

For path B $$Q_B = \Delta E_B + W_B$$

Combining two cycles

$$\left(\sum W\right)_{cycle} = \left(\sum Q\right)_{cycle}$$

or $$W_A + W_B = Q_A + Q_B$$

i.e. $$Q_A - W_A = W_B - Q_B$$

i.e. $$\Delta E_A = -\Delta E_B$$

Similarly, had the system returned from state 2 to state 1 by path C instead of path B, then

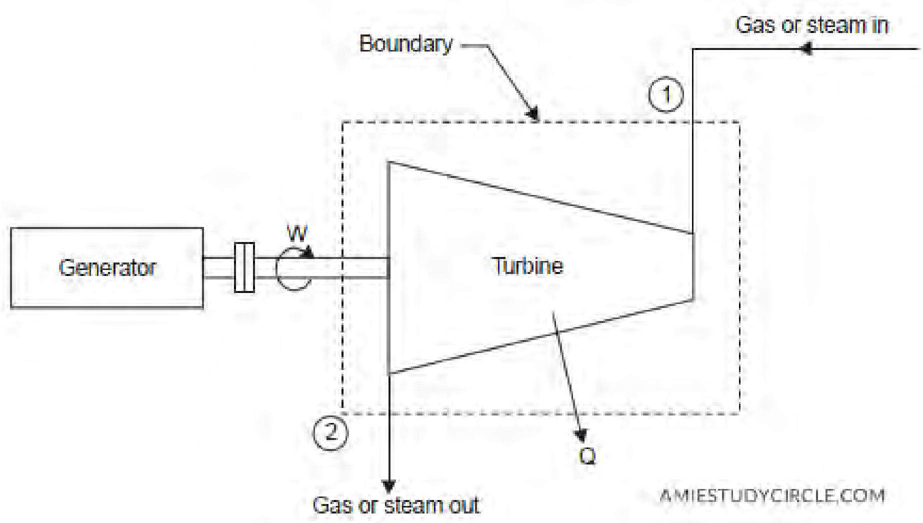
$$\Delta E_A = -\Delta E_C$$

Also $$\Delta E_B = \Delta E_C$$

Therefore, it is seen that the change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state. Further, energy has a definite value for every state of the system.

Steam or Gas Turbine

In a steam or gas turbine steam or gas is passed through the turbine and part of its energy is converted into work in the turbine. This output of the turbine runs a generator to produce electricity as shown in given figure. The steam or gas leaves the turbine at lower pressure or temperature.



Applying energy equation to the system.

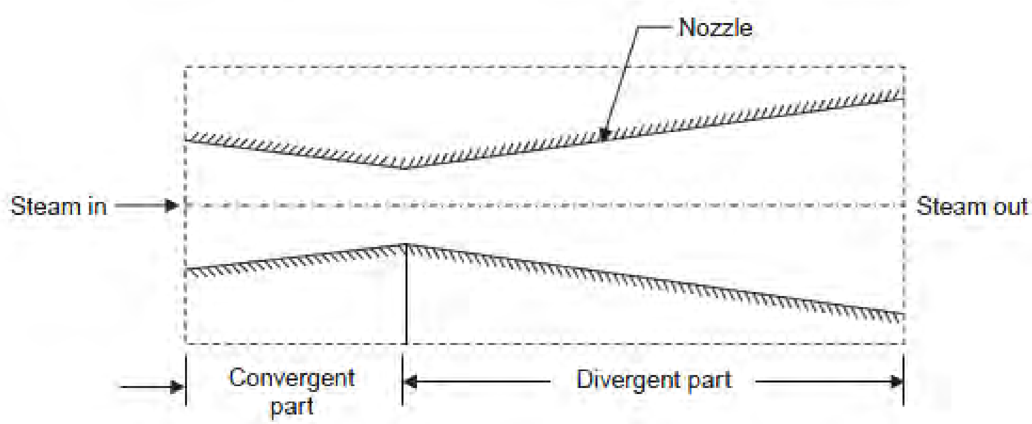
Here, $Z_1 = Z_2 (\Delta Z = 0)$

$$h_1 + \frac{C_1^2}{2} - Q = h_2 + \frac{C_2^2}{2} + W$$

The sign of Q is negative because heat is rejected (or comes out of the boundary). The sign of W is positive because work is done by the system (or work comes out of the boundary).

Steam Nozzle

In case of a nozzle as the enthalpy of the fluid decreases and pressure drops simultaneously the flow of fluid is accelerated. This is generally used to convert the part of the energy of steam into kinetic energy of steam supplied to the turbine.



Above figure shows a commonly used convergent-divergent nozzle.

For this system,

$$\Delta \text{ PE} = 0$$
$$W = 0$$
$$Q = 0$$

Applying the energy equation to the system,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

or $$\frac{C_2^2}{2} - \frac{C_1^2}{2} = h_1 - h_2$$

or $$C_2^2 - C_1^2 = 2(h_1 - h_2)$$

$\therefore$ $$C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)}$$

Here velocity C is in m/s and enthalpy h is in joules.

Example

In a nozzle air at 627°C and twice atmospheric pressure enters with negligible velocity and leaves at a temperature of 27°C. Determine velocity of air at exit, assuming no heat loss and nozzle being horizontal. Take C_p = 1.005 kJ/kg.K for air.

Solution

Applying steady flow energy equation with inlet and exit states as 1, 2 with no heat and work interaction and no change in potential energy.

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

Given that, $C_1 \approx 0$, negligible inlet velocity

$$C_2 = \sqrt{2(h_1 - h_2)}$$

Exit velocity $$C_2 = \sqrt{2C_1(T_1 - T_2)}$$

Given, T_1 = 900 K, T_2 = 300 K

$$C_2 = \sqrt{2(1.005x10^3)(900-300)} = 1098.2 \text{ m/s}$$

Example (AMIE S16, 6 marks)

In a gas turbine unit, the gases flow through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and

400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are 50 m/s and 110 m/s respectively. Calculate :

(i) The rate at which heat is rejected to the turbine, and

(ii) The area of the inlet pipe given that the specific volume of the gases at the inlet is 0.45 m^3/kg.

Solution

Rate of flow of gases, m_1 = 15 kg/s

Volume of gases at the inlet, v = 0.45 m^3/kg

Power developed by the turbine, P = 12000 kW

∴ Work done W = 12000/15 = 800 kJ/kg

Enthalpy of gases at the inlet, h_1 = 1260 kJ/kg

Enthalpy of gases at the outlet, h_2 = 400 kJ/kg

Velocity of gases at the inlet, C_1 = 50 m/s

Velocity of gases at the outlet, C_2 = 110 m/s.

(i) Heat rejected, Q

Using the flow equation,

$$h_1 + \frac{C_1^2}{2} - Q = h_2 + \frac{C_2^2}{2} + W \qquad (1)$$

Kinetic energy at inlet = $\frac{C_1^2}{2} = \frac{50^2}{2} m^2/s^2 = \frac{50\,kgm^3}{2\,s^2kg} = 1250\,Nm/kg = 1.25\,kJ/kg$

Kinetic energy at outlet = $\frac{C_2^2}{2} = \frac{110^2}{2x1000} = 6.05\,kJ/kg$

Substituting these values in eqn. (1), we get

1260 + 1.25 + Q = 400 + 6.05 + 800

∴ Q = – 55.2 kJ/kg

i.e., Heat rejected = + 55.2 kJ/kg = 55.2 × 15 kJ/s = 828 kW.

(ii) Inlet area, A

Using the relation,

$$m = \frac{CA}{v}$$

$$A = \frac{vm}{C} = \frac{0.45x15}{50} = 0.135\,m^2$$

Example

(i) Is the first law of thermodynamics applicable to irreversible processes?

(ii) State the important consequences of the first law of thermodynamics.

Solution

(i) The first law of thermodynamics is applicable to all types of processes-reversible as well as irreversible.

(ii) The first law of thermodynamics leads to the following important consequences.

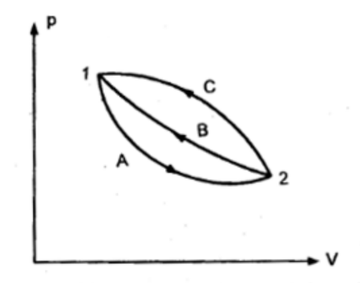
1. heat interaction is a path function.
2. Energy is a property of a thermodynamic system.
3. The energy of an isolated system is conserved.
4. A perpetual motion machine of the first kind is impossible.

Example (AMIE S16, 6 marks)

Show that a change in internal energy of a system is independent of the path followed by the system.

Solution

Consider a system which changes its state from state 1 to state 2 by following path, and returns from state 2 to state 1 by following the path B as shown in following figure.



$$Q_A = \Delta E_A + W_A \quad (1)$$

and path B

$$Q_B = \Delta E_B + W_B \quad (2)$$

The processes A and B together constitute a cycle, for which

$$\Sigma W_{cycle} = \Sigma Q_{cycle}$$

or $W_A + W_B = Q_A + Q_B$

or $Q_A - W_A = W_B - Q_B \quad (3)$

From eq (1, 2, 3)

$$\Delta E_A = -\Delta E_B$$

Similarly had the system returned from state 2 to state 1 following the path C, instead of path B, then

$$\Delta E_A = -\Delta E_C$$

Therefore $\Delta E_B = \Delta E_C$

which means the change in internal energy is **independent of the path** followed and therefore internal energy is a **thermodynamic property**.

Example (AMIE S2008, 5 marks)

Show that the first law of thermodynamics implies that a Perpetual Motion Machine of the First Kind (PMMFK) is impossible

Solution

We know that the macroscopic modes of energy can be convened from one form to the other and work can be obtained. However, the microscopic modes of energy cannot be readily converted into macroscopic modes of energy. An important application of thermodynamics is to devise means of converting the microscopic modes of energy into the macroscopic modes of energy. For this purpose heat engines which work cyclically arc devised. The first law of thermodynamics when applied to a cyclic process gives

$$\oint (dQ - dW) = 0$$

or $$\oint dQ = \oint dW$$

or $$Q = W$$

where Q is the net heat interaction and W is the net work delivered.

An imaginary device which would deliver work continuously without absorbing energy as heat is called a **Perpetual Motion Machine** of the First Kind (PMMFK). A perpetual motion machine of the first kind has to operate on a cycle to deliver work continuously. If the device does not absorb any energy as heat in a cycle, then $\int dQ = Q = 0$. Then the first law of thermodynamics tells that for such a device $\int dW = W = 0$. Thus, the first law of thermodynamics implies that it is impossible to devise a PMMFK. A PMMFK violates the first law of thermodynamics.

Examples (AMIE W06, S10, 15, 16, 6 marks)

State the limitations of the first law of thermodynamics with the help of examples involving heat and work interactions.

Solution

We know that kinetic energy and potential energy are interconvertible and the macroscopic modes of energy (KE and PE) can be readily converted into work. The conversion of microscopic modes of energy (that is energy associated with the random molecular motion of the matter) or internal energy into work requires a device called heat engine. Is it possible for the complete conversion of internal energy into work in a heat engine? Or is it possible to devise a heat engine, the efficiency (defined as the ratio of the net work done to the energy absorbed) of which is equal to one? Is it possible to transfer energy as heat spontaneously from a body at a lower temperature to a body at a higher temperature? All the above questions which deal with work and heat interactions cannot be answered by the application of the first law of thermodynamics and hence they reflect the limitations of the first law thermodynamics.

Example

A stationary mass of gas is compressed without friction from an initial state of 0.3 m^3 and 0.105 MPa to a final state of 0.15 m^3 and 0.105 MPa, the pressure remaining constant during the process. There is a transfer of 37.6 kJ of heat from the gas during the process. How much does the internal energy of the gas change?

Solution

First law for a stationary system in a process gives

$$Q = \Delta U + W$$

or $$Q_{1-2} = U_2 - U_1 + W_{1-2}$$

$$W_{1-2} = \int_{V_1}^{V_2} pdV = p\left(V_2 - V_1\right) \quad (1)$$

$$= 0.105\ (0.15 - 0.30)\text{MJ}$$

$$= -15.75 \text{ kJ}$$

$$Q_{1-2} = -\ 37.6 \text{ kJ}$$

$\therefore$ Substituting in equation (1)

$$-\ 37.6 \text{ kJ} = U_2 - U_1 - 15.75 \text{ kJ}$$

$$U_2 - U_1 = -\ 21.85 \text{ kJ}$$ **Ans.**

The internal energy of the gas decreases by 21.85 kJ in the process.

Example

In a closed system

(a) 17 kJ of heat is added to the system whilst 8 kN-m of work is performed by the system, determine the change in internal energy of the system.

(b) 16 kN-m work is performed by the system and no heat transfer occurs, determine the change in internal energy.

(c) 105 kJ of heat is transferred to the system and 20 kN-m of work is performed on the system, determine the change in internal energy,

(d) 21 kJ of heat is added and no change in internal energy occurs, determine the work done by the system.

Solution

We can apply the energy equation as obtained from the first law of thermodynamics applied to a closed system, i.e.

$$Q = W + \Delta U$$

Heat Q should be taken positive if it is supplied to the system, and negative if rejected by the system. Work W should be taken positive if it is developed by the system and negative if supplied to the system.

(a) $Q = W + \Delta U$

$\Rightarrow$ 17000 = 8000 + ΔU (J = 1 N-m/J)

$\therefore\ \Delta U = \dfrac{17000 - 8000}{1000} = 9$ kJ (increase)

(b) $Q = W + \Delta U$

$\Rightarrow 0 = 16000 + \Delta U \Rightarrow \Delta U = -16\,\text{kJ}$ (decrease)

(b) $Q = W + \Delta U$

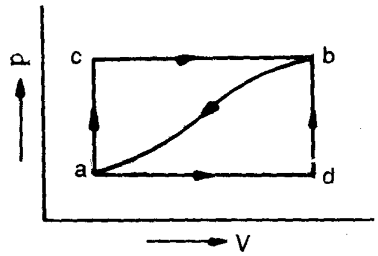
$\Rightarrow$ 105 = - 20 + ΔU ΔU = 105 + 20 = 125 kJ (increase)

(d) $Q = W + \Delta U\ \Delta U$

$\Rightarrow$ $21 = \dfrac{W}{J} + 0$ W = 21 kN-m (work developed by the system)

Example

When a system is taken from state a to state b, in figure, along path acb, 84 kJ of heat flow into the system, and the system does 32 kJ of work. (a) How much will the heat that flows into the system along path adb be, if the work done is 10.5 kJ? When the system is returned from b to a along the curved path, the work done on the system is 21 kJ. Does the system absorb or liberate heat, and how much of the heat is absorbed or liberated? (c) If U_a = 0 and U_d = 42 kJ, find the heat absorbed in the processes ad and db.



Solution

$Q_{acb} = 84$ kJ

$W_{acb} = 32$ kJ

We have

$Q_{acb} = U_b - U_a + W_{acb}$

$\therefore$ $U_b - U_a = 84 - 32 = 52$ kJ **Ans.**

(a) $Q_{adb} = U_b - U_a + W_{adb}$

$= 52 + 10.5$

$= 62.5$ kJ **Ans.**

(b) $Q_{b-a} = U_a - U_b + W_{b-a}$

$= -52 -21$

$= - 73$ kJ **Ans.**

The system liberates 73 kJ of heat.

(c) $W_{adb} = W_{ad} + W_{db} = W_{ad} = 10.5$ kJ

$Q_{ad} = U_d - U_a + W_{ad}$

$42 - 0 + 10.5 = 52.5$ kJ

Now $Q_{adb} = 62.5 \text{ kJ} = Q_{ad} + Q_{db}$

$\therefore$ $Q_{db} = 62.5 - 52.5 = 10$ kJ **Ans.**

Example

A piston an cylinder machine contains a fluid system which passes through a complete cycle of four processes. During a cycle, the sum of all heat transfers is -170 kJ. The system completes 100 cycles per min. Completer the following table showing the method for each item, and compute the net rate of work output in kW.

Process	Q (kJ/min)	W (kJ/min)	ΔE(kJ/min)
a - b	0	0	-
b - c	21,000	-	-
c - d	- 2,100	-	- 36,600
d - a	-	-	-

Solution

Process a - b:

$Q = \Delta E + W$

$0 = \Delta E + 2170$

$\therefore \quad \Delta E = - 2170$ kJ/min

Process b - c:

$Q = \Delta E + W$

$21,000 = \Delta E + 0$

$\therefore \quad \Delta E = - 21,000$ kJ/min

Process c - d:

$Q = \Delta E + W$

$- 2100 = - 36,600 + W$

$\therefore \quad W = 34,500$ kJ/min

Process d - a:

$$\sum_{cycle} Q = -170 kJ$$

The system completes 100 cycles/min.

$\because \quad Q_{ab} + Q_{bc} + Q_{cd} + Q_{da} = - 17000$kJ/min

$0 + 21,000 - 2100 + Q_{da} = -17,000$

$\therefore \quad Q_{da} = -35,900$ kJ/min

Now $\oint dE = 0$, since cyclic integral of any property is zero.

$\therefore \quad \Delta E_{a-b} + \Delta E_{b-c} + \Delta E_{c-d} + \Delta E_{d-a} = 0$

$- 2170 + 21,000 + 36,600 + \Delta E_{d-a} = 0$

$\therefore \quad \Delta E_{d-a} = 17,770$ kJ/min

$\therefore \quad W_{d-a} = Q_{d-a} - \Delta E_{d-a}$

$= -35,900 - 17,770 = - 53,670$ kJ/min

The table becomes

Process	Q (kJ/min)	W (kJ/min)	ΔE(kJ/min)
a - b	0	2170	- 2170
b - c	21,000	0	21,000
c - d	- 2,100	34,500	- 36,600
d - a	- 35,900	- 53,670	17,770

Since $\sum_{cycle} Q = \sum_{cycle} W$

Rate of work output = - 17,000 kJ/min = -283.3 kW **Ans.**

Problem

A gas undergoes a thermodynamic cycle consisting of three processes beginning at an initial state where p_1 = 1 bar, V_1 = 1.5 m^3 and U_1 = 512 kJ. The processes are as follows:

(i) *Process 1-2: compression with pV = constant to p_2 = 2 bar, U_2 = 690 kJ*

(ii) *Processes 2-3: W_{23} = 0, Q_{23} = - 150 kJ, and*

(iii) *Process 3-1: W_{31} = +50 kJ. Neglecting KE and PE change, determine the heat interaction Q_{12} and Q_{31}.*

Answer: 74 kJ, 22 kJ

Problem (AMIE Summer 2013, 6 marks)

A mass of 8 kg gas expands within a flexible container so that the p-v relationship is of the form $pv^{1.2}$ = const. The initial pressure is 1000 kPa and the initial volume is 1 m^3. The final pressure is 5 kPa. If specific internal energy of the gas decreases by 40 kJ/kg, find the heat transfer in magnitude and direction.

Answer: +2615 kJ

Problem

A mixture of gases expands at constant pressure from 1 MPa, 0.03 m^3 to 0.06 m^3 with 84 kJ positive heat transfer. There is no work other than that done on a piston. Find ΔE for the gaseous mixture.

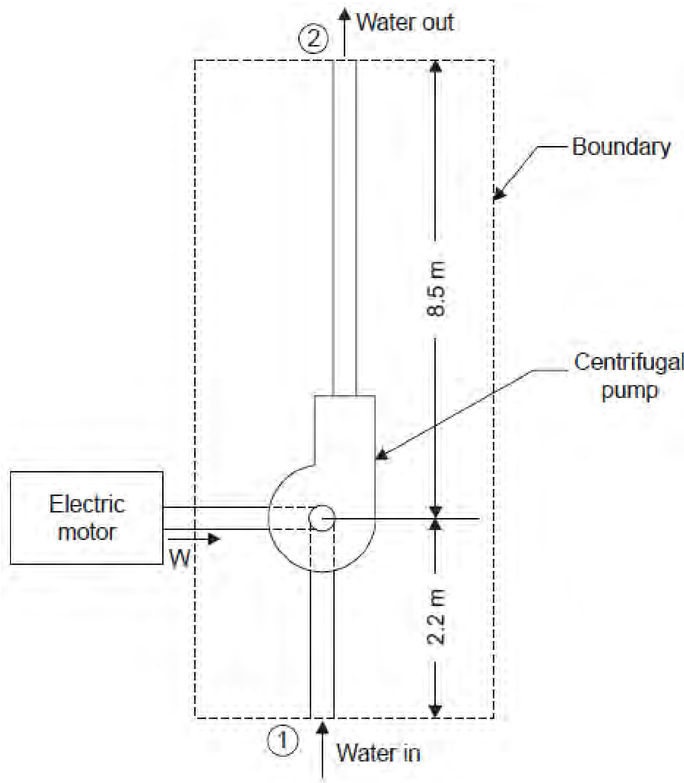
Answer: 54 kJ

AMIE (AMIE W08, S18, 9 marks)

A centrifugal pump delivers 50 kg of water per sec. The inlet and outlet pressures are 1 bar and 4.2 bar, respectively. The suction is 2.2 m below the centre of the pump and delivery is 8.5 m above the centre of the pump. The suction and delivery pipe diameters are 20 cm and 10 cm, respectively. Determine the capacity of electric motor to run the pump.

Solution

Figure



Given data

Quantity of water delivered by the pump, m_w = 50 kg/s

Inlet pressure, p_1 = 1 bar = 1×10^5 N/m^2

Outlet pressure, p_2 = 4.2 bar = 4.2×10^5 N/m^2

Suction-below the centre of the pump = 2.2 m

Delivery-above the centre of the pump = 8.5 m

Diameter of suction pipe, d_1 = 20 cm = 0.2 m

Diameter of delivery pipe, d_2 = 10 cm = 0.1 m

Capacity of electric motor

Steady flow energy equation is given by

$$m_w\left(u_1 + p_1v_1 + \frac{C_1^2}{2} + Z_1g\right) + Q = m_w\left(u_2 + p_2v_2 + \frac{C_2^2}{2} + Z_2g\right) + W \qquad (1)$$

Considering the datum from suction 1, as shown

$Z_1 = 0$, $Z_2 = 8.5 + 2.2 = 10.7$ m

$u_2 - u_1 = 0$; $Q = 0$

Thus eqn. (i) reduces to

$$W = m_w\left[(p_1v_1 - p_2v_2) + (Z_1 - Z_2)g + \left(\frac{C_1^2 - C_2^2}{2}\right)\right] \qquad (2)$$

As water is incompressible fluid

$$v_2 = v_1 = v = 1/\rho = 1/1000$$

The mass flow through inlet and exit pipe is given by

$$m_w = \frac{\pi}{4}d_1^2C_1\rho = \frac{\pi}{4}d_2^2C_2\rho$$

∴ $$50 = \frac{\pi}{4}(0.2)^2C_1x1000$$

∴ C_1 = 1.59 m/s

Similarly C_2 = 6.37 m/s

Substituting the values in (2)

$$W = 50\left[\left(1x10^5x\frac{1}{1000} - 4.2x10^5x\frac{1}{1000}\right) + (0 - 10.7)x9.81 + \left(\frac{1.59^2 - 6.37^2}{2}\right)\right]$$

= 22.2 kW

SPECIFIC HEAT

Specific heat of a substance is the amount of heat that must be added to unit mass of the substance to raise the temperature through 1°C. The symbol "c" will be used for specific heat.

$$c(\text{or }C) = \frac{Q}{m.\Delta t}J/kgK$$

Gases have two specific heats, namely, specific heat at constant volume and specific heat at constant pressure.

Specific heat at constant volume (C_v): The amount heat required in kJ to raise the temperature of 1 kg of the gas through 1 K at constant volume is known as specific heat at constant volume. There is no work of expansion as the gas volume remains constant, and , therefore, all the heat supplied is used to increase its internal energy.

Specific heat at constant pressure (C_p): The amount of heat required in kJ to raise the temperature of 1 kg of the gas through 1 K at constant pressure is known as specific heat at constant pressure.

When the gas is heated at constant pressure, it gets expanded and moves the piston through a distance L., therefore in addition to the heat required to increase the kinetic energy of the molecule, further heat must be added to perform the work of moving the piston through a distance L. Therefore, the specific heat of a gas at constant pressure is always greater than the specific heat at constant volume by an amount equivalent to expansion work.

The ratio of two specific heats, C_p and C_v of any gas is assumed to a constant and this is expressed by the symbol γ (gamma),

$$\gamma = \frac{C_p}{C_p} = \frac{1.004}{0.715} = 1.4 \text{ for all}$$

The relation between two specific heats is given by

$$C_p - C_v = R$$

Where, R is known as characteristic gas constant.

Example

The specific heat of a gas at constant volume is 3.15 kJ/kg-^oC and the ratio of specific heats γ = 1.66 for the same gas. If 1.5 kg of this gas is heated from 50^oC to 350^oC at constant pressure, determine heat supplied to the gas in kJ.

SOLUTION

We know that

$$\gamma = \frac{C_p}{C_v}$$

$$\therefore \quad C_p = \gamma \times C_v = 1.66 \times 3.15 = 5.23 \text{ kJ/kg-}^oC$$

Heat supplied to the gas at constant pressure

$$= m \,.\, C_p \,.\, (T_2 - T_1) = 1.5 \times 5.23 \times (350 - 50) = 2353 \text{ kJ}$$ **Ans.**

SECOND LAW OF THERMODYNAMICS

According to the first low of thermodynamics, from 1 N-m of work, one joule of heat can be obtained and 1 N-m of work can be obtained from 1 Joule of heat if so converted. But the general observation is that while work can be completely converted to heat, but it is not possible to convert heat completely into work. It becomes necessary for any heat engine to reject some of the heat it receives during the cycle to the surroundings even if an ideal engine is considered. The limitation of the first law is that it does not simply unidirectional phenomenon concerned with energy conversion.

It is observed in the nature that heat by itself never flows from an object at a lower temperature to one at a higher temperature, in the same way that a river never flows by itself uphill. These observations are the basis of the second law of thermodynamics. The second law of thermodynamics states the possibility and extent of transformation of heat into work. The different statements of second law of thermodynamics are given below.

Kelvin Plank Statement

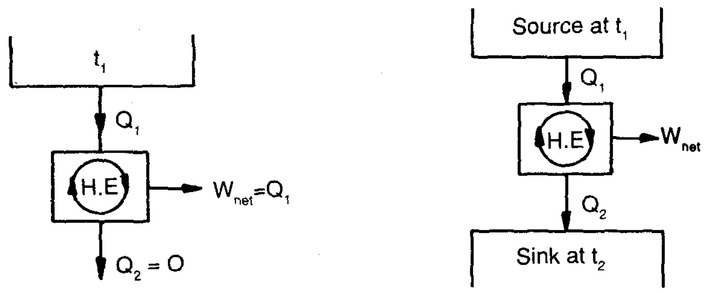
It states that it is impossible to construct and engine working is a cyclic process, whose sole effect is the conversion of all the heat supplied to it into an equivalent amount of work.

$$\eta = \frac{W_{net}}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Practically $W_{net} < Q_1$. Hence *an engine can not be 100% efficient*.

If $Q_2 > 0$, there will always be *heat rejection*.

If $Q_2 = 0$, then $\eta = 1$ (100%), then *we get perpetual motion machine (PMM). PMM is impossible*. Figure (a) shows a PMM.



(a) **(b)**

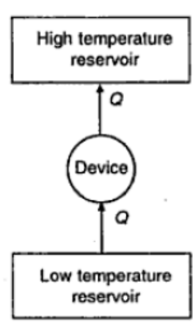
A heat engine has, therefore, to exchange heat with two thermal energy reservoirs at two different temperatures to produce net work in a complete cycle (figure b). Motive power can be produced till there is *no* difference of temperature.

Clausius' Statement

It is impossible to construct a device working in cyclic process whose sole effect is transfer of heat from a body at a lower temperature to a body of higher temperature.

Heat cannot flow of itself from a body at a lower temperature to a body at a higher temperature. Some work must be expended to achieve this.

A schematic of the device which is impossible to devise according to the Clausius statement of the second law of thermodynamics is shown in figure.



We know that the transfer of energy as heat from a high temperature body to a low temperature body occurs spontaneously. The Clausius statement of the second law of thermodynamics denies the possibility of self reversal of such a spontaneous process. In other words the second law of thermodynamics dictates the direction of a spontaneous process. The coefficient of performance of the device shown in given figure is given by $COP = Q/W = Q/0 = \infty$. Thus the second law of thermodynamics implies that the COP of a heat pump or refrigerator cannot be infinity.

Example (AMIE W06, 08, 16, S08, 15, 17, 8 marks)

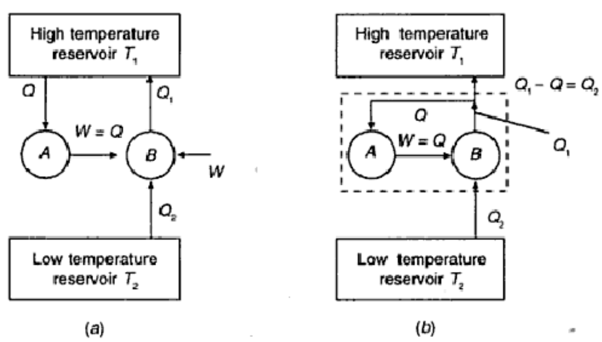
State the second law of thermodynamics as stated by Kelvin-Plank and Clausius. Show the equivalence of the above two statements.

Solution

Equivalence of the Two statements: The equivalence is established by showing that any device that violates Clausius statement leads to violation of Kelvin Planck statement and vice versa.

Violation of the Kelvin-Planck statement leads to violation of Clausius statement of the second law of thermodynamics.

To prove that violation of Kelvin-Planck statement implies the violation of the Clausius statement, let us suppose that the Kelvin-Planck statement of the second law of thermodynamics is *incorrect*. that is, it is possible to devise a cyclically operating device A which absorbs energy *Q* as heat from a thermal reservoir at high temperature T_1 and delivers an equivalent amount of work W (W = Q) in a cycle as shown in figure.



Now let us consider another cyclically operating device B which absorbs energy Q_2 as heat from a low temperature reservoir at T_2 ($T_2 < T_1$) and rejects energy Q_1 as heat to a high temperature reservoir at T_1 when work W is done on the device in one cycle as shown in figure (a). The device B is aided and hence it is not in violation of the Clausius statement. The work done on the device B is given by $W = Q_1 - Q_2$. Let us consider the combination of the devices A and B as shown in figure (b).The work delivered by the device A is used to operate the device B and part of the energy Q rejected by device B is absorbed by device A.

$$W = Q$$

and $W = Q_1 - Q_2$

or $Q = Q_1 - Q_2$

or $Q_1 - Q = Q_2$

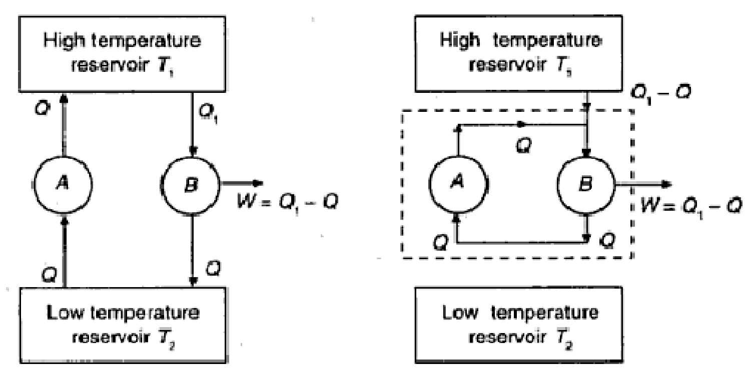
The combination of the devices A and B is shown in figure (b). The combined device absorbs energy Q_2 as heat from a low temperature reservoir at T_2 and rejects energy Q_2 ($Q_2 = Q_1$ - Q) as heat to the high temperature reservoir at T_1 while it is unaided by any external agency. That is, it is possible to devise a self acting cyclically operating device which transfers energy as heat from a low temperature body to a high temperature body. This is in *violation* of the Clausius statement of the second Law of thermodynamics. Thus, violation of the Kelvin-Planck statement leads to violation of the Clausius statement.

Violation of Clausius statement leads to violation of the Kelvin-Planck statement of the second law of thermodynamics.

To prove that violation of Clausius statement leads to violation of Kelvin-Planck statement, let us assume that the Clausius statement of the second law of thermodynamics, is incorrect. That is, it is possible to devise a self acting (or externally unaided) cyclically operating device A which transfers energy Q as heat from a low temperature reservoir at T_2 to a high temperature reservoir at T_1 ($T_1 > T_2$) as shown in figure (a). Now. let us consider another device B which absorbs energy Q_1 as heat from a high temperature reservoir at T_1, does work. W on the surroundings and rejects energy Q as heat to the low temperature reservoir at T_2, as shown in figure (a). The device B does not violate the Kelvin-Planck statement of the second law of thermodynamics. The work delivered by device B is given by

$$W = Q_1 - Q$$

Now, let us combine the devices A and B such that the energy Q rejected as heat at temperature T_2, by device B is directly fed to the device A and the energy Q rejected as heat at temperature T_1 by device A is directly absorbed by device B. The difference in the energy (Q_1 - Q) is absorbed by device B from the high temperature reservoir at T_1 and the work delivered by device B is W($W = Q_1$ - Q) as shown in figure (b).

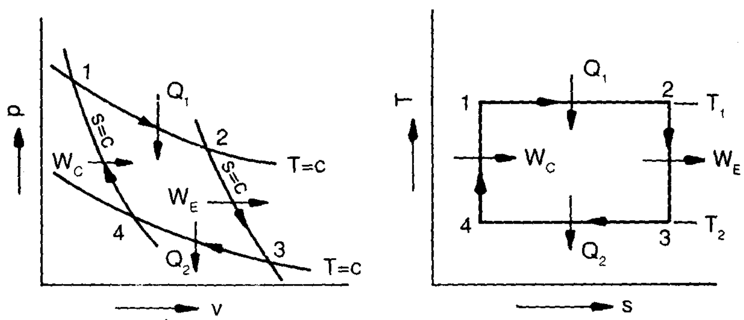


(a) (b)

The combined device effectively absorbs energy (Q_1 - Q) as heat from a single thermal reservoir and delivers an equivalent amount of work W (W = Q_1 - Q) in violation of the Kelvin-Planck statement of the second law of thermodynamics. *Thus violation of Clausius statement leads to violation of Kelvin-Planck statement. Since, the violation of Kelvin-Planck statement leads to violation of Clausius statement and violation of Clausius statement leads to violation of Kelvin-Planck statement, these two statements of the second law of thermodynamics are equivalent.*

Carnot Cycle

The second law of thermodynamics states that *only a part of the heat supplied to the heat engine can be converted into work.* Naturally the next step is to examine the practical means for doing so and analyse their effectiveness. Carnot postulated that the most efficient engine is a *reversible* engine and be devised such an engine working on a cycle known as *Carnot cycle*. All the constituent processes of the cycle are reversible processes as shown in figure given below on p-v and T-s diagram.



Carnot cycle

1. The process 1-2 represents isothermal expansion of the working fluid in the cylinder.
2. The process 2-3 represents isentropic expansion of the working fluid.
3. The process 3-4 represents isothermal compression, and
4. The process 4-1 represents isentropic compression till the fluid reaches its original condition completing the cycle.

The efficiency of the cycle is given by

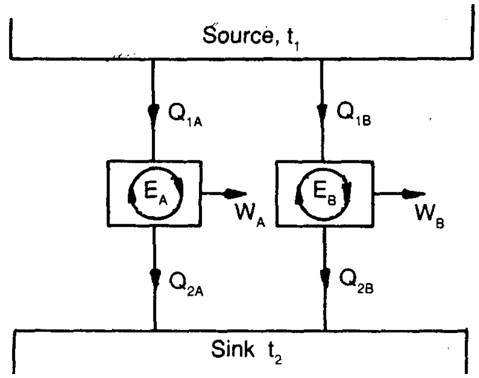
$$\eta = \frac{W_{net}}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Carnot efficiency is the highest possible efficiency of a heat engine. The efficiency is also independent of the type of the fluid used. Therefore it is considered as a standard of comparison for heat engine performance.

Carnot Theorem

It states that of all heat engines operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine.

Proof. Let two heat engines E_A and E_B operate between the given source at temperature t_1 and the given sink at temperature t_2 as shown in figure (a).



Let E_A be any heat engine and E_B be any reversible heat engine. We have to prove that the efficiency of E_B is more than that of E_A. Let us assume that this is not true and $\eta_A > \eta_B$. Let the rates of working of the engines be such that

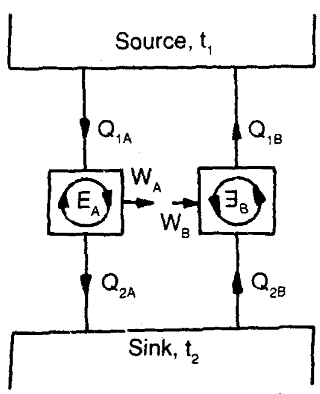
$$Q_{1A} = Q_{1B} = Q_1$$

Since $$\eta_A > \eta_B$$

$$\frac{W_A}{Q_{1A}} > \frac{W_B}{Q_{1B}}$$

$\therefore$ $$W_A > W_B$$

Now, let E_B be reversed. Since E_B is a reversible heat engine, the magnitudes of heat and work transfer quantities will remain the same, but their directions will be reversed, as shown in figure (b).



E_B is reversed

Since $W_A > W_B$, some part of W_A (equal to W_B) may be fed to drive the reversed heat engine $\exists_B$.

Since $Q_{1A} = Q_{1B} = Q_1$, the heat discharged by $\exists_B$ may be supplied to E_A. The source may, therefore, be eliminated (figure c). The net result is that E_A and $\exists_B$ together constitute a heat engine which, operating in a cycle, produces net work W_A - W_B, while exchanging heat with a single reservoir at t_2. This violates the Kelvin-Planck statement of the second law. Hence the assumption that $\eta_A > \eta_B$ is wrong.

Therefore $\eta_B \geq \eta_A$

Efficiency of The Reversible Heat Engine

The efficiency of a reversible heat engine in which heat is received solely at T_1 is found to be

$$\eta_{rev} = \eta_{max} = 1 - \left(\frac{Q_2}{Q_1}\right)_{rev} = 1 - \frac{T_2}{T_1}$$

or $$\eta_{rev} = \frac{T_1 - T_2}{T_1}$$

The COP (Coeff. of Performance) of a refrigerator is given by

$$(COP)_{refr} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{(Q_1/Q_2) - 1}$$

For a reversible refrigerator, using

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$[COP_{refr}]_{rev} = \frac{T_2}{T_1 - T_2}$$

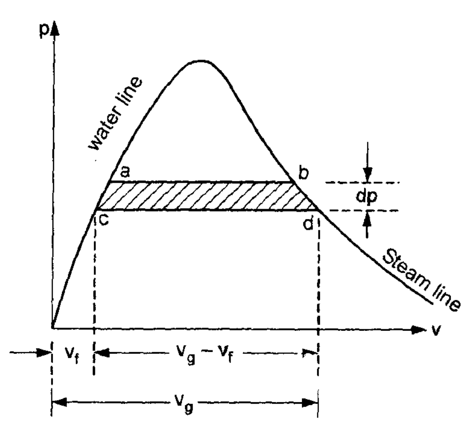
Similarly, for a reversible heat pump

$$[COP_{H.P.}]_{rev} = \frac{T_1}{T_1 - T_2}$$

CLAPEYRON'S THEOREM

It is derived from Carnot theorem.

Carnot efficiency is given by $(T_1 - T_2)/T_1$ and for a small pressure drop dp, and the corresponding temperature drop dT, the Carnot efficiency is given by dT_s/T_s. Here T_s is saturation temperature. In evaporation the heat supplied is latent heat.



From p-v diagram of above figure,

work done = area a-b-c-d = $(v_g - v_f)\, dp$

It is also equal to

heat supplied x efficiency = $h_{fg}.\frac{dT_s}{T_s}$

Equating the two

$$(v_g - v_f)dp = h_{fg}.\frac{dT_s}{T_s}$$

or $$\frac{dp}{dT_s} = \frac{h_{fg}}{T_s(v_g - v_f)}$$

This is Clapeyron's theorem.

Example (AMIE Summer 2011, 6 marks)

The efficiency of a Carnot engine can be increased either by decreasing the sink temperature while keeping the source temperature constant or by increasing the source temperature while keeping the sink temperature constant. Which one of the above two possibilities is more effective?

Solution

Let T_1 = source temperature; T_2 = sink temperature

Carnot cycle efficiency

$$\eta_c = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \text{ g}$$

$$\left(\frac{\partial \eta_c}{\partial T_2}\right)_{T_1} = -\frac{1}{T_1}$$

and $$\left(\frac{\partial \eta_c}{\partial T_1}\right)_{T_2} = -T_2(-T_1)^{-2}\frac{T_2}{T_1^2}$$

$\therefore$ $$\frac{-\left(\frac{\partial \eta_c}{\partial T_2}\right)_{T_1}}{\left(\frac{\partial \eta_c}{\partial T_1}\right)_{T_2}} = \frac{1}{T_1}.\frac{T_1^2}{T_2} = \frac{T_1}{T_2}$$

Since $T_1 > T_2$, $T_1/T_2 < 1$

$\therefore$ Decrease of sink temperature T_2, $\left(\partial \eta_c / \partial T_2\right)$ is more effective than increasing the source temperature T_1.

Example

Distinguish between reversible and irreversible processes and give some examples of irreversible processes.

Solution

A process is said to be reversible if both the system and its surroundings can be restored to their respective initial states by reversing the direction of the process. If a process does not satisfy the above criterion, it is an irreversible process.

Examples of irreversible processes are :

- Expansion or compression with finite pressure difference.
- Energy transfer as heat with finite temperature difference.
- Free expansion of a gas.
- Mixing of non identical gases.
- Mixing of matter at different states.
- Motion with friction.
- Viscous fluid flow.
- Spontaneous chemical reactions.

Example

What is the importance of reversible processes in engineering thermodynamics?

Solution

A reversible process is an idealization. This is a concept which can be approximated closely at times by actual devices, but never followed.

The presence of friction, inelasticity and electrical resistance makes the processes irreversible. These elements can be reduced but cannot be completely eliminated. The presence of these elements of irreversibility makes a process irreversible. Since the real processes occur when these elements of irreversibility are present, the reversible process is a limiting process toward which all actual processes may approach in performance. The reversible processes deliver maximum work in engines and require minimum work in devices such as refrigerators, compressors etc. A thermodynamic analysis based on idealized reversible processes provides the limiting performance of the devices, against which the actual performance can be compared. This in turn provides an opportunity to improve the performance of the devices by reducing the sources of irreversibility.

Example (AMIE W2009, 2 marks)

Give the criteria of reversibility, irreversibility and impossibility of a thermodynamic cycle.

Solution

Since the second law of thermodynamics distinguishes between reversible and irreversible processes, one can use the following criterion to identify a reversible process. If a process can proceed in either direction without violating the second law of thermodynamics, then it is a reversible process. Suppose a process is proceeding in one direction and the assumption of reversibility of the process leads to a violation of the second law of thermodynamics, then the process can be called irreversible.

Example (AMIE S09, 3 marks)

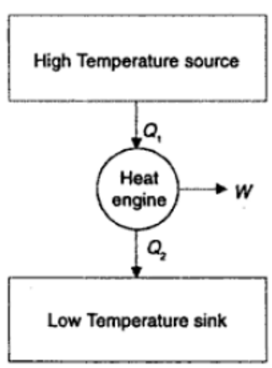
What is meant by a heat engine and what are its characteristics?

Solution

A heat engine is an energy conversion device. It is a cyclically operating device and its primary objective is to convert the energy received as heat into work. It employs a working fluid which undergoes cyclic change. The working fluid absorbs energy as heat from a source and rejects energy as heat to a sink. The characteristics of a heat engine are :

- It is a cyclically operating device.
- Its primary purpose is to convert energy absorbed as heat into work.
- It absorbs energy as heat from a high temperature source.
- It rejects energy as heat to a low temperature sink.
- It delivers some net work.

A heat engine can be represented as shown in figure.



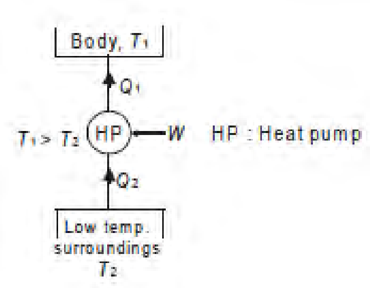
Example (AMIE S16, 17, 6 marks)

Explain (i) heat pump (ii) refrigerator. How is COP of a heat pump related to the COP of a refrigerator?

Solution

Heat Pump

Heat pump refers to a device used for extracting heat from a low temperature surroundings and sending it to high temperature body, while operating in a cycle. In other words *heat pump maintains a body or system at temperature higher than temperature of surroundings*, while operating in cycle. Block diagram representation for a heat pump is given below:



As heat pump transfers heat from low temperature to high temperature, which is non spontaneous process, so external work is required for realizing such heat transfer. Heat pump shown picks up heat Q_2 at temperature T_2 and rejects heat Q_1 for maintaining high temperature body at temperature T_1.

For causing this heat transfer heat pump is supplied with work W as shown.

As heat pump is not a work producing machine and also its objective is to maintain a body at higher temperature, so its performance can't be defined using efficiency as in case of heat engine. Performance of heat pump is quantified through a parameter called coefficient of performance (C.O.P). Coefficient of performance is defined by the ratio of desired effect and net work done for getting the desired effect.

COP = desired effect/Net work done

For heat pump :

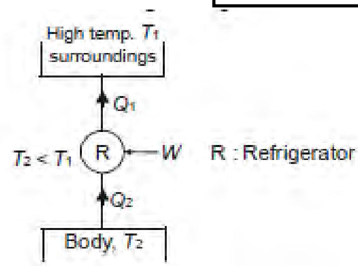
Net work = W

Desired effect = heat transferred Q_1 to high temperature body at temperature, T_1.

$$COP = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

Refrigerator

Refrigerator is a device similar to heat pump but with reverse objective. It maintains a body at temperature lower than that of surroundings while operating in a cycle. Block diagram representation of refrigerator is shown in given figure.



Refrigerator also performs a non spontaneous process of extracting heat from low temperature body for maintaining it cool, therefore external work W is to be done for realizing it.

Block diagram shows how refrigerator extracts heat Q_2 for maintaining body at low temperature T_2 at the expense of work W and rejects heat to high temperature surroundings.

Performance of refrigerator is also quantified by coefficient of performance, which could be defined as:

$$COP_{ref} = \frac{desired\ effect}{net\ work} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

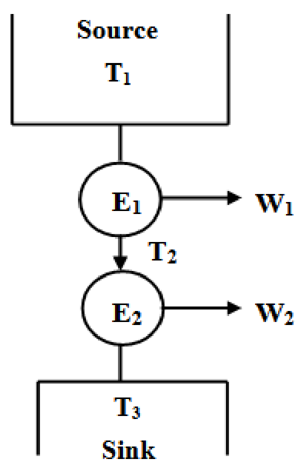
COP values of heat pump and refrigerator can be interrelated as:

$$COP_{hp} = COP_{ref} + 1$$

Example

A reversible engine receives heat from a reservoir at 700°C and rejects the heat at temperature T_2. A second reversible engine receive heat reject by the first engine and reject to the sink at 37°C. Determine the temperature T_2 if (a) both the engines give same thermal efficiency (b) both the engines develop same power.

Solution



(a) For equal efficiency condition

$$\frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2}$$

$$\frac{973 - T_2}{973} = \frac{T_2 - 310}{T_2}$$

Gives on solving, T = 549.2 K

(b) The work-output of any reversible engine is proportional to the temperature difference between source and sink,

$W_1 = K . (T_1 - T_2)$

$W_2 = K . (T_2 - T_3)$

For the given condition.

$W_1 = W_2$

$T_1 - T_2 = T_2 - T_3$

$$\Rightarrow \quad T_2 = \frac{T_1 - T_3}{2} = \frac{973 + 310}{2} = 641.5 \text{ K}$$

Example

An inventor claims that his petrol engine operating between temperatures of 2000°C and 600°C will produce 1 H.P.hr., consuming 120 gm of petrol having 46025 kJ/kg calorific value. Check the validity of his claim.

Solution

1 H.P. hr = 735.5 x 60 x 60 = 2647.8 kJ

The efficiency of the engine of the inventor

$$= \frac{\text{Output}}{\text{Input}} = \frac{2647.8}{\frac{120}{1000} \text{x}\, 46025} = 0.48 = 48 \ \%$$

The maximum efficiency of any engine working between the given temperature limits is Carnot efficiency.

$$\therefore \quad \text{Maximum } \eta = \frac{T_1 - T_2}{T_1} = \frac{2000 - 600}{(2000 + 273)} = 0.616 = 61.6 \ \%$$

As the efficiency claimed is less than the maximum possible, his claim is valid.

Example

An inventor claims to have designed a heat engine which absorbs 1 kJ of energy as heat at 727^{0}C and delivers 0.6 kJ of work when the ambient temperature is 27^{0}C. Would you agree with this claim?

Solution

The maximum possible efficiency of a heat engine operating between two thermal reservoirs is given by

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1000} = 0.7$$

Efficiency claimed by the inventor,

$$\eta_{claim} = \frac{W}{Q_1} = \frac{0.6}{1} = 0.6$$

The claimed efficiency (0.6) is less than the maximum possible efficiency (0.7) and hence it is feasible to devise such an engine.

Example

An inventor claims that his engine absorbs 300 kJ of energy from a thermal reservoir at 325 K and delivers 75 kJ of work. The inventor also states that his engine has two heat rejections: 125 kJ to a reservoir at 300 K and 100 kJ to a reservoir at 275 K. Check the validity of his claim.

Solution

The validity of the proposed engine is checked on the basis whether it satisfies or not the first and second laws of thermodynamics. The engine is theoretically feasible if both the laws are satisfied. The device becomes impossible if any of the two laws is violated.

$$\oint \delta Q = 300 - 125 - 100 = 75 \text{ kJ}$$

$$\oint \delta W = 75 \; kJ$$

Since $\oint \delta Q = \oint \delta W$ (according to first law)

Applying Clausius theorem

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = \frac{300}{325} - \frac{125}{300} - \frac{100}{275} = 0.416 > 0$$

The Clausius theorem requires that $\oint \frac{\delta Q}{T} \le 0$ and as the proposed engine gives $\oint \frac{\delta Q}{T} > 0$, *the claim of the inventor is not acceptable.*

Example

Which is the more effective way to increase the efficiency of a Carnot engine to increase T_1, keeping T_2 constant; or to decrease T_2, keeping T_1 constant?

Solution

Let T_2 be decreased by ΔT with T_1 remaining the same

$$\eta_1 = 1 - \frac{T_2 - \Delta T}{T_1}$$

If T_1 is increased by the same ΔT with T_1 remaining the same

$$\eta_2 = 1 - \frac{T_2}{T_1 + \Delta T}$$

Then $$\eta_1 - \eta_2 = \frac{T_2}{T_1 + \Delta T} - \frac{T_2 - \Delta T}{T_1}$$

$$= \frac{(T_1 - T_2)\Delta T + (\Delta T)^2}{T_1(T_1 + \Delta T)}$$

Since $T_1 > T_2$

$\therefore$ $(\eta_1 - \eta_2) > 0$

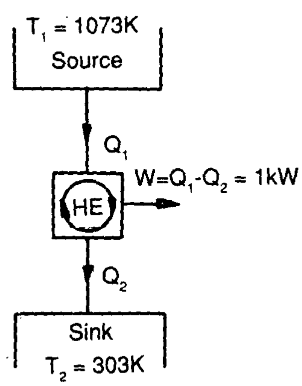
The more efficient way to increase the cycle efficiency is to decrease T_2.

Example

A cyclic heat engine operates between a source temperature of 800^0C and a sink temperature of 30^0C. What is the least rate of heat rejection per kW net output of the engine?

Solution

For a reversible engine, the rate of heat rejection will be minimum.



We have $\eta_{max} = \eta_{rev} = 1 - \frac{T_2}{T_1} = 1 - \frac{30+273}{800+273} = 0.718$

Now $\frac{W_{net}}{Q_1} = \eta_{max} = 0.718$

$\therefore$ $Q_1 = \frac{1}{0.718} = 1.392 kW$

Now $Q_2 = Q_1 - W_{net} = 1.392 - 1 = 0.392 kW$

This is the least rate of heat rejection.

Example

A domestic food freezer maintains a temperature of -15^{0}C. The ambient air temperature is 30^{0}C. If heat leaks into the freezer at the continuous rate of 1.75 kJ/s what is the least power necessary to pump this heat out continuously?

Solution

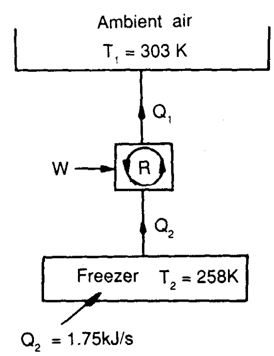
Freezer temperature

$$T_2 = -15 + 273 = 258K$$

Ambient air temperature

$$T_1 = 30 + 273 = 303K$$

The refrigerator cycle removes heat from the freezer at the same rate at which heat leaks into it (see figure).



Example

A reversible heat engine operates between two reservoirs at temperatures of 600^{0}C and 40^0. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40^{0}C and - 20^0. The heat transfer to the heat engine is 2000 kJ and the net work output of the combined engine refrigerator plant is 360 kJ.

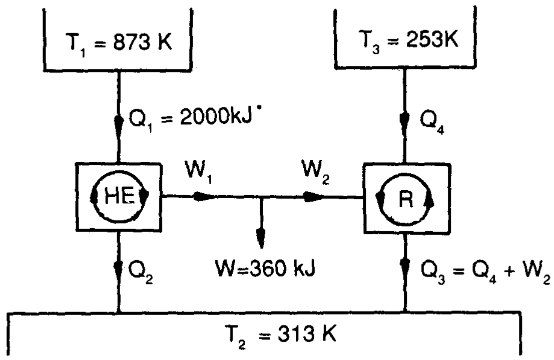
(a) Evaluate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40^{0}C.

(b) Reconsider (a) given that the efficiency of the heat engine and the COP of the refrigerator are each 40% of their maximum possible values.

Solution

(a) Maximum efficiency of the heat engine cycle (see figure) is given by

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} = -0.358 = 0.642$$



Again $\frac{W_1}{Q_1} = 0.642$

$\therefore$ W_1 = 0.642 x 2000 = 1284 kJ

Maximum COP of the refrigerator cycle

$$(COP)_{max} = \frac{T_3}{T_2 - T_3} = \frac{253}{313 - 253} = 4.22$$

Also $COP = \frac{Q_4}{W_2} = 4.22$

Since $W_1 - W_2 = W$ = 360 kJ

$\therefore$ $W_2 = W_2 - W$ = 1284 -360 = 924 kJ

$\therefore$ Q_4 = 4.22 x 924 = 3899 kJ

$\therefore$ $Q_3 = Q_4 + W_2$ = 924 + 3899 = 4823 kJ

$Q_2 = Q_1 + W_1$ = 2000 - 1284 = 716 kJ

Heat rejection to the 40^0C reservoir

$= Q_2 + Q_3$ = 716 + 4823 = 5539 kJ Ans. (a)

(b) Efficiency of the actual heat engine cycle

$\eta = 0.4\ \eta_{max}$ = 0.4 x 0.642

$\therefore$ W_1 = 0.4 x 0.642 x 2000 = 513.6 kJ

$\therefore$ W_2 = 513.6 - 360 = 153.6 kJ

COP of the actual refrigerator cycle

$$COP = \frac{Q_4}{W_2} = 0.4 x 4.22 = 1.69$$

Therefore

$Q_4 = 153.6 \text{ x } 1.69 = 259.6$ kJ Ans. (b)

$Q_3 = 259.6 + 153.6 = 413.2$ kJ

$Q_2 = Q_1 - W_1 = 2000 - 513.6 = 1486.4$ kJ

Heat rejected to the 40^0C reservoir

$= Q_2 + Q_3 = 413.2 + 1486.4 = 1899.6$ kJ Ans. (b)

Example

A reversible engine is used for only driving a reversible refrigerator. Engine is supplied 2000 kJ/s heat from a source at 1500 K and rejects some energy to a low temperature sink. Refrigerator is desired to maintain the temperature of 15ºC while rejecting heat to the same low temperature sink. Determine the temperature of sink if total 3000 kJ/s heat is received by the sink.

Solution

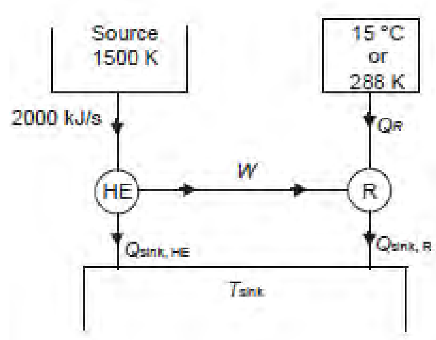
Let temperature of sink be T_{sink} K.

Given: $Q_{sink, HE} + Q_{sink, R} = 3000$ kJ/s

Since complete work output from engine is used to run refrigerator so,

$$2000 - Q_{sink, HE} = Q_{sink, R} - Q_R$$

$$Q_R = 3000 - 2000 = 1000 \text{ kJ/s}$$



Also for engine $\dfrac{2000}{1500} = \dfrac{Q_{sink,HE}}{T_{sink}} \Rightarrow Q_{sink,hE} = \dfrac{4}{3}T_{sink}$

For refrigerator $\dfrac{Q_R}{288} = \dfrac{Q_{sink,R}}{T_{sink}} \Rightarrow Q_{sink,R} = \dfrac{1000}{288}T_{sink}$

Substituting $Q_{sink,HE}$ and $Q_{sink,R}$ values.

$$\frac{4}{3}T_{sink} + \frac{1000T_{sink}}{288} = 3000$$

$\therefore$ $T_{sink} = 624.28\,K$

Temperature of sink = 351.28 ºC

Example

A house requires 2×10^5 kJ/h for heating in winter. Heat pump is used to absorb heat from cold air outside in winter and send heat to the house. Work required to operate the heat pump is 3×10^4 kJ/h. Determine :

(i) Heat abstracted from outside ;

(ii) Co-efficient of performance.

Solution

(i) Heat requirement of the house, Q1 (or heat rejected)

$= 2 \times 10^5$ kJ/h

Work required to operate the heat pump,

$W = 3 \times 10^4$ kJ/h

Now, $Q_1 = W + Q_2$

where Q_2 is the heat abstracted from outside.

$\therefore\ 2 \times 10^5 = 3 \times 10^4 + Q_2$

Thus $Q_2 = 2 \times 10^5 - 3 \times 10^4$

$= 200000 - 30000 = 170000$ kJ/h

Hence, heat abstracted from outside = 170000 kJ/h. (Ans.)

(ii)
$$COP_{heat\ pump} = \frac{Q_1}{Q_1 - Q_2} = \frac{2x10^5}{2x10^5 - 170000} = 6.66$$

Hence, co-efficient of performance = 6.66.

Note. *If the heat requirements of the house were the same but this amount of heat had to be abstracted from the house and rejected out, i.e., cooling of the house in summer, we have*

$$COP_{refrigerator} = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W} = \frac{170000}{3x10^4} = 5.66$$

Thus the same device has two values of C.O.P. depending upon the objective.

Problem

In a winter season when outside temperature is –1ºC, the inside of house is to be maintained at 25ºC. Estimate the minimum power required to run the heat pump of maintaining the temperature. Assume heating load as 125 MJ/h.

Answer: 3.02 kW

Problem

A household refrigerator is maintained at a temperature of 2^0C. Every time the door is opened, warm material is placed inside, introducing an average of 420 kJ, but making only a small change in the temperature of the refrigerator. The door is opened 20 times a day, and the refrigerator operates at 15% of the ideal COP. The cost of work is 32 paisa per kWh. What is the monthly bill for this refrigerator? The atmosphere is at 30^0C.

Answer: Rs. 15.20

Problem

A heat pump working on the Carnot cycle takes in heat from a reservoir at 5^0C and delivers heat to a reservoir at 60^0C. The heat pump is driven by a reversible heat engine which takes in heat from a reservoir at 840^0C and rejects heat to a reservoir at 60^0C. The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from the 5^0C reservoir, determine (a) the rate of heat supply from 840^0C source (b) the rate of heat rejection to the 60^0C sink.

Answer: (a) 47.61 kW (b) 34.61 kW

Problem

Two reversible heat engines A and B are arranged in series. A rejecting heat directly to B. Engine A receives 200 kJ at a temperature of 421^0C from a hot source, while engine B is in communication with a cold sink at a temperature of 4.4^0C. If the work output of A is twice that of B, find (a) the intermediate temperature between A and B (b) the efficiency of each engine, and (c) the heat rejected to the cold sink.

Answer: 143.4^0C, 40% & 33.5%, 80 kJ

Example

Two Carnot engines I and II operate in series between a high temperature reservoir at 1027^0C and a low temperature reservoir al 27^0C. The engine I absorbs energy from the high temperature reservoir and rejects energy to a reservoir at temperature T, The engine II receives energy from the reservoir at T and rejects energy to the low temperature reservoir The amount of energy absorbed by engine II from the reservoir at T is the same as that rejected by engine I to the reservoir at T, If engines I and II are found to have the same efficiency, determine the temperature T. If engine I receiver 100 kJ energy as heat from the high temperature reservoir, calculate the work delivered by engine I and engine II.

Solution

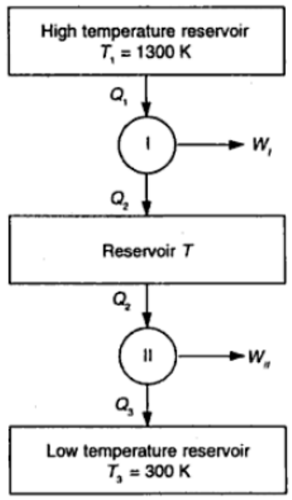
A schematic of engines I and II is shown below.

$$\eta_1 = 1 - \frac{T}{T_1}; \eta_2 = 1 - \frac{T_3}{T}$$

$$\eta_1 = \eta_2$$

$$1 - \frac{T}{T_1} = 1 - \frac{624.5}{1300} = 0.5196$$

$$W_1 = \eta_1 Q_1 = 0.5196 x 100 = 51.96\ kJ$$



$$\frac{Q_2}{Q_1} = \frac{T}{T_1}$$

$$Q_2 = \frac{T}{T_1} Q_1 = \frac{624.5}{1300} x 100 = 48.038\ kJ$$

$$W_2 = \eta_2 Q_2 = 0.5196\ x\ 48.038 = 24.96\ kJ$$

Example (AMIE S2006, 8 marks)

A reversible engine works between three thermal reservoirs A, B and C. The engine absorbs an equal amount of heat from the thermal reservoirs A and B kept at temperatures T_A and T_B, respectively, and rejects heat to the thermal reservoir C kept at temperature T_C. The efficiency of the engine is α times the efficiency of the reversible engine, which works between the two reservoirs A and C. Prove that

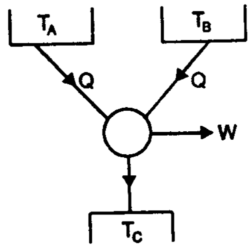
$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\frac{T_A}{T_C}$$

Solution

Refer following figure.

Let the engine be supplied Q units of heat from each of the thermal reservoirs A and B. When operating between reservoirs A and C

$$\eta_{th} = \frac{T_A - T_C}{T_A}$$



Since Q units of heat is supplied from reservoir A, the work output will be

$$Q\left(\frac{T_A - T_C}{T_A}\right)$$

Likewise when the operation is between reservoir B and C, the work output will be

$$Q\left(\frac{T_B - T_C}{T_B}\right)$$

$\therefore$ For the given engine

Total heat output = 2Q

Total work output

$$= Q\left(\frac{T_A - T_C}{T_A}\right) + Q\left(\frac{T_B - T_C}{T_B}\right)$$

Thermal efficiency

$$= \frac{Q\left(\frac{T_A - T_C}{T_A}\right) + Q\left(\frac{T_B - T_C}{T_B}\right)}{2Q}$$

$$= \frac{1}{2}\left[1 - \frac{T_C}{T_A} + 1 - \frac{T_C}{T_B}\right]$$

$$= \frac{1}{2}\left[2 - \frac{T_C}{T_A} - \frac{T_C}{T_B}\right]$$

As per given condition

$$\frac{1}{2}\left[2 - \frac{T_C}{T_A} - \frac{T_C}{T_B}\right] = \alpha\left[\frac{T_A - T_C}{T_A}\right] = \alpha\left(1 - \frac{T_C}{T_A}\right)$$

Multiplying both sides by $2\frac{T_A}{T_C}$, we get

$$2\frac{T_A}{T_C} - 1 - \frac{T_A}{T_B} = 2\alpha\frac{T_A}{T_C} - 2\alpha$$

or $$\frac{T_A}{T_B} = (2\alpha - 1) + 2\frac{T_A}{T_C} - 2\alpha\frac{T_A}{T_C}$$

$$= (2\alpha - 1) + 2\frac{T_A}{T_C}(1-\alpha)$$

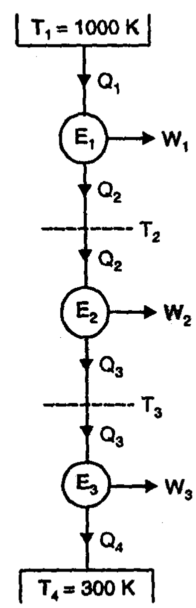
$$= 2(1-\alpha)\frac{T_A}{T_C} + (2\alpha - 1)$$

Example

Three Carnot engines E_1, E_2 and E_3 operate between temperatures of 1000 K and 300 K. Make calculations for the intermediate temperatures if the work produced by the engines are in the ratio of 4:3:2.

Solution

See following figure for schematic arrangement of three Carnot engines in series.



For Engine E_1

$$\eta_1 = \frac{W_1}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{1000 - T_2}{1000}$$

$$Q = W_1 + Q_2$$

$\therefore$ $$\frac{W}{W_1 + Q_2} = \frac{1000 - T_2}{1000}$$

$$Q_2 = W_1\frac{T_2}{1000 - T_2}$$

For Engine E_2

$$\eta_2 = \frac{W}{Q_2} = \frac{T_2 - T_3}{T_2}$$

substituting the value of Q_2 as obtained above

$$\eta_2 = \frac{W_2}{W_1}\left(\frac{1000 - T_2}{T_2}\right) = \frac{T_2 - T_3}{T_2}$$

Since $$\frac{W_2}{W_1} = \frac{3}{4}$$

we get $$\frac{3}{4}(1000 - T_2) = T_2 - T_3$$

or $$750 - (3/4)T_2 = T_2 - T_3$$

i.e. $$T_2 = \frac{4}{7}(750 + T_3) \quad (1)$$

Also $$Q_2 = Q_3 + W_2$$

$$\therefore \quad \frac{W}{Q_3 + W_2} = \frac{T_2 - T_3}{T_2}$$

i.e. $$Q_3 = W_2 \frac{T_3}{T_2 - T_3}$$

For Engine 3

$$\eta_3 = \frac{W_3}{Q_3} = \frac{T_3 - 300}{T_3}$$

substituting the value of Q_3 as outlined above

$$\frac{W_3}{W_2}\left(\frac{T_2 - T_3}{T_3}\right) = \frac{T_3 - 300}{T_3}$$

Since $$\frac{W_3}{W_2} = \frac{2}{3}$$

we get $$\frac{2}{3}(T_2 - T_3) = T_3 - 300 \quad (2)$$

Solving (2) and (3)

T_2 = 689 K and T_3 = 455.80 K

Problem

Three Carnot heat engines are arranged in series. The first engine takes 4000 kJ of heat from a source at 2000 K and delivers 1800 kJ of work; the second and third engines deliver 1200 kJ and 500 kJ of work respectively. Make calculations for the exhaust temperature of the second and third Carnot engine.

Answer: 250 K, 500 K

ASSIGNMENT

Q.1. (AMIE W10, 6 marks): Explain the concept of macroscopic and microscopic viewpoint as applied to study of thermodynamics.

Q.2. (AMIE S06, 5 marks): Explain the following terms in the context of thermodynamics: (i) System, (ii) Surrounding, (iii) Universe, (iv) State, (v) Properties.

Q.3. (AMIE W08, 11, 10, S17, 18, 6 marks): What is a thermodynamic system? Differentiate between open system, closed system and isolated system.

Q.4. (AMIE W05, S15, 4 marks): Define properly and mention its main characteristic in relation to a cyclic process.

Q.5. (AMIE S12, 3 marks): Under what conditions is the work done equal to $\int pdv$?

Q.6. (AMIE W09, S12, 6 marks): Show that energy is a property of a system. What are the modes in which energy is stored in a system? What is the difference between the standard symbols of E and U?

Q.7. (AMIE S13, 6 marks): Explain the difference between energy in transit and energy in storage. What is the energy per unit mass for a (i) non-flow system, and (ii) flow system?

Hint: Non flow means closed system.

Q.8. (AMIE W05, 4 marks): In what respects are the heat and work interactions (i) similar and (ii) dissimilar?

Q.9. (AMIE S07, 16, 5 marks): Define heat and work. Which are the characteristics common to both work and heat?

Q.10. (AMIE S06, 2 marks): The two modes of energy transfer are work and heat. Does the mode of energy transfer depend upon the choice of a system? Support your answer with the help of an example.

Q.11. (AMIE W09, 2 marks) Does heat transfer inevitably cause a temperature rise?

Q.12. (AMIE S10, 2 marks): Distinguish between thermodynamic work and heat transfer.

Q.13. (AMIE S06, 08, 5 marks): State and explain the conditions to be satisfied for thermodynamic equilibrium.

Q.14. (AMIE W07, 2 marks): Distinguish between steady slate and equilibrium.

Q.15. (AMIE W05, 4 marks): Define property and mention its main characteristic in relation to a cyclic process.

Q.16. (AMIE W06, S15, 4 marks): Distinguish clearly between the following, giving examples wherever necessary:

(i) Closed system and open system;

(ii) Heat and work;

(iii) Point functions and path functions;

(iv) Enthalpy and internal energy.

Q.17. (AMIE W12, 2 marks): Differentiate between heat and internal energy.

Q.18. (AMIE W12, 2 marks): Explain system approach and control volume approach in the analysis of a flow process.

Q.19. (AMIE W06, 4 marks): Define the first law of thermodynamics for a closed system. Explain the limitations of the first law.

Q.20. (AMIE W10, 6 marks): Derive an expression for the first law of thermodynamics of an open system.

Q.21. (AMIE S10, W12, 6 marks): Explain the first law of thermodynamics for a change of state and prove that energy is a property.

Q.22. (AMIE S15, 4 marks): Explain the first law of thermodynamics as referred to a closed systems undergoing a cyclic change.

Q.23. (AMIE W16, 17, 8 marks): State the first law of thermodynamics for a closed system undergoing a cycle. Apply the first law for a steady flow process by taking a real life example. How does Bernoulli's equation compare with steady flow energy equation?

Q.24. (AMIE W08, 4 marks): What do you mean by the "Perpetual Motion Machine of first kind (PMM-1)"?

Q.25. (AMIE W97): Explain the two statements of second law of thermodynamics.

Q.26. (AMIE S10, 4 marks): State the statements of second law of thermodynamics as pertaining to a heat engine and a refrigerator.

Q.27. (AMIE S13, 5 marks): What is a reversible process? How is a reversible process only a limiting process, never to be attained in practice?

Q.28. (AMIE S11, 13, 5 marks): Establish the equivalence of Kelvin-Planck and Clausius statements.

Q.29. (AMIE S16, 6 marks): State and prove Clausius inequality.

Q.30. (AMIE S13, 17, 5 marks): To produce network in a thermodynamic cycle, a heat engine has to exchange heat with two thermal reservoirs. Explain.

Hint: Kelvin – Planck statement

Q.31. (AMIE S16, 6 marks): State Kelvin-Plank and Clausius statements of second law of thermodynamics. Show that the violation of Clausius statement leads to the violation of Kelvin-Plank statement.

Q.32. (AMIE W05, 6 marks): State and prove Carnot theorem.

Q.33. (AMIE S05, 4 marks): What is Carnot cycle? What are the four processes which constitute the cycle? Explain a reversible isothermal and a reversible adiabatic process.

Q.34. (AMIE W07, 2 marks): Explain why Carnot cycle cannot be realized in practice?

Q.35. (AMIE W17, 6 marks): Explain the limitation of Carnot cycle in second law of thermodynamics.

Q.36. (AMIE W11, 8 marks): Derive an expression for the efficiency of a reversible heat engine.

Hint: Carnot cycle is reversible.

Q.37. (AMIE W05, 12, 4 marks): Derive Clapeyron's equation. What is its use and limitations?

Q.38. (AMIE S05, W09, 6 marks): Show that the efficiency of a reversible heat engine operating between two given constant temperatures is the maximum.

Q.39. (AMIE S12, 5 marks): How is a reversible process only a limiting process, never to be attained in practice? What do you understand by internal and external irreversibilities?

Q.40. (AMIE S05, W12, 6 marks): Prove that no heat engine can be more efficient than a reversible heat engine when both of them are operating between the same thermal reservoirs.

Q.41. (AMIE S10, 6 marks): Prove that the efficiency of Carnot engine is higher compared to that of an irreversible heat engine, both working between same temperature limits.

Q.42. (AMIE S07, 2 marks): How is the COP of a heat pump related to the COP of a refrigerator?

Q.43. (AMIE W09, 5 marks): Explain the following statements:

(i) To produce net work in a thermodynamic cycle, a heat engine has to exchange heat with two thermal reservoirs.

(ii) All spontaneous processes are irreversible.

Q.44. (AMIE W11, 12 marks): 3 kg of an ideal gas is expanded from a pressure 7 bar and volume 1.5 m^3 to a pressure 1.4 bar and volume 4.5 m^3. The change in internal energy is 525 kJ. The specific heat at constant volume for the gas is 1.047 kJ/kg-K. Calculate (i) gas constant; (ii) change in enthalpy; and (iii) initial and final temperatures.

Answer: 0.8376 kJ/kg K, 945 kJ, 250.7 K

Q.45. (AMIE W12, 4 marks): Nitrogen gas at 300 K, 101 kPa and 0.1 m^3 is compressed slowly in an isothermal process to 500 kPa. Calculate the work done during the process.

Answer: -16.155 kJ (compression work)

Q.46. (AMIE W05, 12 marks): A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship p = a + bv, where a and b are constants. The initial and final pressures are 1000 kPa and 200 kPa, respectively and the corresponding volumes are 0.20 m^3 and 1.20 m^3. The specific internal energy of the gas is given by u = 1.5pv - 85 kJ/kg

where p is in kPa and v is in m^3/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.

Answer: 660 kJ, 503 .25 kJ

Q.47. (AMIE S10, 4 marks): The pressure volume relation for a non-flow reversible process is P = (8 - 4V) bar, where V is in m^3. If 130 kJ of work is supplied to the system, calculate the final pressure and volume of the system. Take the initial volume as 0.5 m^3.

Answer: 6.812 bar, 0.297 m^3

Q.48. (AMIE W10, 8 marks): A fluid system undergoes a non-flow frictionless process following the pressure volume relations as p = (5/ v) + 1.5 where p is in bar and v is in m^3. During the process, the volume changes from 0.15 m^3 to 0.05 m^3 and the system rejects 45 kJ of heat. Determine (i) change in internal energy, and (ii) change in enthalpy.

Answer: 519.3 kJ, 504.35 kJ

Hint: Enthalpy = $H_2 - H_1 = (U_2 + p_2V_2) - (U_1 + p_1V_1)$

Q.49. (AMIE W10, 8 marks): At the inlet to a certain nozzle, the enthalpy of the fluid passing is 2800 kJ/kg and the velocity is 50 m/s at the discharge end. The enthalpy is 2600 kJ/kg; nozzle is horizontal; and there is a negligible heat loss from it.

(i) Find the velocity at the exit of the nozzle.

(ii) If the inlet area is 900 cm2 and specific volume at inlet is 0.187 m^3/kg, find the mass flow rate.

(iii) If the specific volume at the nozzle exit is 0.498 m^3/kg, find the exit area of nozzle.

Answer: 634.43 m/s, 24.06 kg/s, 188.89 cm^2

Hint: In part (ii) and (iii), apply continuity equation, $mv_1 = C_1A_1$ and $mv_2 = C_2A_2$

Q.50. (AMIE S13, 8 marks): At the inlet to a pipeline, the condition of steam is p = 4MPa, t = 400 °C, h = 3213.6 kJ/kg and v = 0.073 m^3/kg. At the discharge end, the conditions are found to be p = 3.5 MPa, t = 390 °C, h = 3202.6 kJ/kg, and v = 0.084 m^3/ kg. If there is a heat loss of 8.5 kJ/ kg from the pipeline, calculate the steam flow rate.

Answer: $1339d^2$ kg/s; d is diameter in m.

Q.51. (AMIE S12, 17, 10 marks): In a gas turbine, the gas enters at the rate of 5 kg/s with a velocity of 50 m/s and enthalpy of 900 kJ/kg and leaves the turbine with a velocity of 150 m/s and enthalpy of 400 kJ/kg. The loss of heat from the gases to the surroundings is 25 kJ/kg. Assume for gas, R = 0.285 kJ/kgK and C_p = 1.004 kJ/kg-K, and the inlet conditions to be at 100 kPa and 27 °C. Determine the power output of the turbine and the diameter of the inlet pipe.

Answer: 2325 kJ/s (kW), 33 cm

Q.52. (AMIE W05, 12 marks): A reversible heat engine receives heat from a high temperature reservoir at T_1 K and rejects heat to a low temperature sink of 800 K. A second reversible engine receives the heat rejected by the first engine at 800 K and rejects to a cold reservoir at 280 K. Make calculations for temperature T_1 (i) for equal thermal efficiencies of the two engines (ii) for the two engines to deliver the same amount of work.

Answer: 2285.7 K, 1320 K

Q.53. (AMIE W12, 8 marks): A reversible heat engine, operating between thermal reservoirs at 300 °C and 30 °C drives a reversible refrigerator which refrigerates a space at -15 °C and delivers heat to a thermal reservoir at 30 °C. The heat input to the heat engine is 1900 kJ and there is a net work output from the combined plant (heat engine and refrigerator) of 290 kJ. Determine the heat transfer to the refrigerant and the total heat transfer to the 30 °C thermal reservoir.

Answer: 3470.3 kJ, -5080.3 kJ

Q.54. (AMIE S09, 6 marks): A heat engine receives heat from a source at 1200 K at a rate of 500 kJ/s and rejects the waste beat to a medium at 300 K. The power output of the heat engine in 180 kW. Determine the reversible power and irreversible rate for this process.

Answer: 375 kJ/s (kW), 195 kW

Q.55. (AMIE W09, 7 marks): Two reversible heat engines A and B arc arranged in series A rejecting heat directly to B. Engine A receives 200 kJ at a temperature of 421 ^{0}C from a hot source, while engine B is in communication with a cold sink at a temperature of 4.4 °C. If the work output of A is twice that of B. find the (i) intermediate temperature between A and B, (ii) efficiency of each engine, and (iii) heat rejected to the cold sink.

Answer: 143.27^{0}C, 40%, 33.36%, 80 kJ

Q.56. (AMIE S10, 8 marks): A combination of three Carnot engines, A, B and C, working in a series, operate between temperatures 1000 K and 300 K. Calculate the intermediate temperature, if the amount of work produced by the engines is in the proportion of 5:4:3.

Answer: 708.3 K, 475.02 K

Q.57. (AMIE W07, S17, 6 marks): A Carnot engine operates between two reservoirs at the temperature of T_1 K and T_2 K. The work output of the engine is 0.6 times that of heat rejected. Given that difference in temperature between the source and sink is 200 °C. Calculate the source temperature, sink temperature, and thermal efficiency.

Answer: 533.3 K, 333.3 K, 37.5%

Q.58. (AMIE W08, 10 marks): A Carnot heat engine draws heat from a reservoir at temperature T_1 and rejects the heat to another reservoir at temperature T_3. The Carnot forward cycle engine derives a Carnot reversed cycle engine or Carnot refrigerator which absorbs the heat from reservoir at temperature T_2 and reject the heat to a reservoir at temperature T_3 if the higher temperature T_1 = 600 K and lower temperature T_2 = 300 K. Determine the (i) temperature T_3 such that the heat supplied to engine Q_1 is equal to the heat absorbed by refrigerator Q_2, and (ii) efficiency of Carnot engine and COP of Carnot refrigerator.

Answer: 400 K

Q.59. (AMIE S05, 10 marks): Two Carnot refrigerators A and B operate in series. The refrigerator A absorbs energy at the rate of 1 kJ/s from a body at 300 K and rejects energy as heat to a body at T. The refrigerator B absorbs the same quantity of energy which is rejected by the refrigerator A from the body at T, and rejects the energy as heat to a body at 1000 K. If both the refrigerators have the same COP, calculate

(i) Temperature T of the body

(ii) The COP of the refrigerators, and

(iii) The rate at which energy is rejected as heat to the body at 1000 K.

Answer: 547.72 K, 1.211, 3.333 kJ/s

Q.60. (AMIE W05, 12 marks): It is desired to compress 10 kg of gas from 1.5 m^3 to 0.3 m^3 at a constant pressure of 1.5 bar. During this compression process, the temperature rises from 20^{0}C to 150^{0}C and the increase in internal energy is 3250 kJ. Calculate the work done, heat interaction and change in enthalpy during the process. Also work out the average value of specific heat at constant pressure.

Answer: -1800 kJ, 1450 kJ, 1450 kJ, 1.1154 kJ/kh^{0}C

Q.61. (AMIE S06, 5 marks): An inventor claims to have designed a heat engine which absorbs 260 kJ of energy as heat from a reservoir at 52 ^{0}C and delivers 72 kJ work. He also states that the engine rejects 100 kJ and 88 kJ of energy to the reservoirs at 27 ^{0}C and 2 °C, respectively. State with justification whether his claim is acceptable or not.

Answer: Not acceptable.

Q.62. (AMIE W10, 6 marks): The first source can supply energy at the rate of 12,000 kJ/min at 320 ^{0}C. The second source can supply energy at the rate of 12,0000 kJ/min at 70 ^{0}C. Which source (1 or 2) would you choose to supply energy to an ideal reversible heat engine to produce large amount of power, if the temperature of the surrounding is 35 ^{0}C?

Q.63. (AMIE W08, 7 marks): 90 kJ of heat is supplied to a system at constant volume. The system rejects 95 kJ of heat at constant pressure and 18 kJ of work is done on it. The system is brought to original state by adiabatic process. Determine (i) adiabatic work, and (ii) values of internal energy at all end states if initial, U_1, value is 105 kJ.

Answer: 195 kJ, 118 kJ, 13 kJ

Q.64. (AMIE W08, 5 marks): A cyclic heat engine operates between the source temperature of 1000 ^{0}C and a sink temperature of 40 °C Find the least rate of heat rejection per kW net output of the engine?

Answer: 0.326 kJ/s

Answer: P_1 = 96.12 kJ/s (kW), P_2 = 204.08 kJ/s (kW). $P_2 > P_1$, we will choose source 2 to supply energy.

Q.65. (AMIE W10, 8 marks): A heat-pump works on a reversed Carnot cycle takes energy from a reservoir maintained at 3 °C and delivered it to another reservoir at a temperature of 77 °C. The heat-pump derives the power for its operation from a reversible engine operating within high and low temperatures of 1077°C and 77 °C, respectively. For 100 kJ/kg of energy supplied to reservoir of 77 ^{0}C, estimate the energy taken from the reservoir at 1077 °C.

Answer: 26.71 kJ

Q.66. (AMIE S13, 6 marks): A heat pump provides 3 x 10^4 kJ/h to maintain a dwelling at 23 °C on a day when the outside temperature is 0 °C. The power input to the heat pump is 4 kW. Determine the COP of the heat pump and compare it with the COP of a reversible heat pump operating between the reservoirs at the same two temperatures.

Answer: 2.083, 12.87

Q.67. (AMIE S12, 12 marks): A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The interior temperature is to be maintained at 20 °C. Heat transfer through the walls and roof is estimated to be 0.525 kW per degree temperature difference between the inside and outside. (i) If the outside temperature in winter is 5 °C, what is the minimum power required to drive the heat pump? (ii) If the power output is the same as in part (i), what is the maximum outer temperature for which the inside can be maintained at 20 °C?

Answer: 0.403 kW, 35^0

Q.68. (AMIE S11, 6 marks): The lowest temperature which has been achieved till date is 0.0014 K. Suppose a sample is to be maintained at that temperature. The energy losses as heat from the sample are estimated at 50J/s and the ambient temperature is 35 ^{0}C. Suppose a reversible heat engine which uses a source at 400 °C and the ambient atmosphere as the sink to drive a reversible heat pump in order to maintain the sample at the required temperature. Determine the power required to operate the heat pump and the ratio of energy absorbed by the heat engine from the source to the energy absorbed by the heat pump from the sample.

Answer: 11 MW, 405.64 x 10^3

Q.69. (AMIE S11, 8 marks): It is proposed to compress air (ideal gas) reversibly from an initial state of 100 kPa and 27 °C to a final state of 500 kPa and 27 °C. Compare the work required for the following processes: (i) Heating at constant volume followed by cooling at constant pressure, (ii) isothermal compression, (iii) adiabatic compression followed by cooling at constant volume. For air, C_v = 20.93 J/molK and C_p = 29.302 J/molK.

Answer: 10046 J/mol, 4042.26 J/mol, 5674 J/mol

Q.70. (AMIE S15, 10 marks): A gas at 65 kPa and 200^0C is heated in a closed rigid vessel till it reaches to 400^0C. Determine the amount of heat required for 0.5 kg of this gas, if internal energy at 200 ^{0}C and 400^0C are 26.6 kJ/kg and 37.8 kJ/kg respectively.

Answer: No work is done being in closed vessel. Heat = 0.5 ($u_1 - u_2$) = 0.5 (37.8 – 26.6) = 5.6 kJ

Q.71. (AMIE S16, 6 marks): Two Carnot engines A and B are connected in series between two thermal reservoirs maintained at 1000 K and 100 K, respectively. Engine A receives 1680 kJ of heat from the high-temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low-temperature reservoir. If engines A and B have equal thermal efficiencies, determine the (i) temperature at which heat is rejected by engine A, (ii) heat rejected by engine B and (iii) work done during the process by engines A and B, respectively.

Answer: 316.2 K; 168 kJ; 1148.7 kJ; 363.3 kJ

Q.72. (AMIE S16, 8 marks): A closed system contains 0.5 kg of air. It expands from 2 bar, 60 °C to 1 bar, 40^0C. During expansion, it receives 2 kJ of heat from a reservoir at 100 °C. Assuming atmospheric conditions to be at 0.95 bar, 30 °C, calculate the (i) maximum work, (ii) work done on atmosphere and (iii) change in availability.

Answer: 28.26 kJ; -19.97 kJ; 8.288 kJ

Q.73. (AMIE W16, 6 marks): Write down 'Carnot Theorem'. It is proposed that solar energy be used to warm a large collector plate. This energy would, in turn, be transferred as heat to a fluid within a heat engine, and the engine would reject energy as heat to the atmosphere.

Experiments indicate that about 1880 kJ/m^2-h of energy can be collected when the plate is operating at 90 ^{0}C. Estimate the minimum collector area that would be required for a plant producing 1 kW of useful shaft power. The atmospheric temperature may be assumed to be 20 °C.

Answer: 10 m^2

Hint: $\eta_{\max} = 1 - \frac{T_2}{T_1}; Q_{\min} = \frac{W}{\eta_{\max}}; A = \frac{Q_{\min}}{Energy\, collected}$

Q.74. (AMIE W17, 4 marks): An inventor claims to have developed an engine capable of delivering a network output of 410 kJ for an energy input by the heat transfer of 1000 kJ. The engine receives heat from the hot gases at a temperature of 500 kJ and discharges heat to the atmosphere at 300 K. Evaluate this claim.

Answer: $\eta_{actual} = 0.41$; $\eta_{max} = 0.40$

The claim is not valid since the actual thermal efficiency is higher than the maximum thermal efficiency.

Q.75. (AMIE W17, 10 marks): A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 27 °C. If the heat removal rate from the refrigerator is 25 kJ/s, determine the COP of the machine and work input required. If the required input to run the pump is developed by a reversible engine which receives the heat at 400 °C and rejects the heat to the atmosphere, then determine the overall COP of the system.

Answer: 5.605

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